1 Introduction

Voltage dividers and potentiometers are passive circuit components that provide a simple way to convert a DC voltage level to another, lower, DC voltage level. Figure 1 shows the electrical circuit of a voltage divider on the left, and a potentiometer on the right.

![Figure 1: A voltage divider on the left, and potentiometer on the right.](image)

A voltage divider consists of two resistors in series with a voltage tap between the two resistors. In the left side of Figure 1, the input voltage, $V_{in}$, is applied across $R_1$ and $R_2$. The output voltage, $V_{out}$, is the voltage drop across $R_2$. $V_{out}$ is less than $V_{in}$ because the total voltage across $R_1$ and $R_2$ must add up to $V_{in}$.

A potentiometer is a voltage divider that allows adjustment of $V_{out}$. Typical potentiometers have sliders or rotary knobs that move a contact called a wiper along the surface of a resistor. As depicted in the right side of Figure 1, the wiper divides a single, fixed resistor into $R_1$ and $R_2$. By sliding the wiper along the fixed resistor, the value of $R_2$ is changed, which allows the output voltage to be adjusted from 0 to $V_{in}$.

Voltage dividers and potentiometers are passive in the sense that they transform $V_{in}$ to $V_{out}$ without a separate source of power. Any power consumed during the transformation comes from the source of the input voltage. In contrast, an active component requires an external source of power to operate. Because voltage dividers and potentiometers are passive, these devices can only decrease the voltage, i.e., $V_{out}$ is always less than $V_{in}$. Boosting the voltage from $V_{in}$ to a higher level $V_{out}$ requires an amplifier, which is an active component.

2 Analysis of a Voltage Divider

Applying Kirchoff’s voltage law and Ohm’s law, we can obtain a simple formula for $V_{out}$ as a function of $V_{in}$, $R_1$ and $R_2$ for the two circuits in Figure 1. The resulting formula applies to either a voltage divider or a potentiometer, the two devices are electrically equivalent.

Figure 2 shows the possible current flows in a voltage divider. For convenience, the bottom nodes of the circuit have been tied to ground. The current flows and other operating characteristics of the voltage divider do not change if the lower nodes are not grounded. The formulas derived in this section do not require the lower nodes to be grounded.
The current flowing out of the voltage divider is called the load current, $I_{\text{load}}$. The current leaving the voltage divider (as opposed to flowing straight to ground) is a load current in the sense that the electrical power is put some use, such as powering a DC motor, an LED, or some other device that consumes electrical energy. The analysis of the voltage divider or potentiometer is separated into two cases: one where $I_{\text{load}}$ is negligible, and one where $I_{\text{load}}$ needs to be included in the analysis.

Apply Kirchoff’s current law to the node between the two resistors in the voltage divider depicted in Figure 2.

$$I_1 = I_{\text{load}} + I_2$$

where $I_1$ and $I_2$ are the currents flowing through $R_1$ and $R_2$, respectively.

The load current is negligible in applications where the purpose of $V_{\text{out}}$ is to provide a reference voltage that controls the operation of another (usually active) electronic device. Potentiometers can be used to provide user input to the operation of an Arduino microcontroller. For example, a potentiometer could adjust a voltage level that an Arduino uses to determine how fast to blink an LED.

### 2.1 Case of Infinite Load Resistance: $I_{\text{load}} = 0$

If the load resistance is infinite, $I_{\text{load}}$ will be zero. There are practical applications where $I_{\text{load}}$ is so small that its affect on the operation of the voltage divider is negligible. Figure 3 shows the current flow in a voltage divider with an infinite load resistance.

If we assume that the load current is negligible, then Equation (1) shows that $I_1 = I_2$. When $I_{\text{load}} = 0$ all the current supplied by the voltage source $V_{\text{in}}$ flows to ground through the series combination of $R_1$ and $R_2$. Let that current be called $I$. In other words, $I_1 = I_2 = I$ as shown in Figure 3.

Apply Ohm’s law to the series combination of $R_1$ and $R_2$ with the common current $I$

$$V_{\text{in}} = IR_{\text{eff}}$$

where $R_{\text{eff}} = R_1 + R_2$ is the effective resistance of $R_1$ and $R_2$ in series. Solve Equation (2) for $I$

$$I = \frac{V_{\text{in}}}{R_1 + R_2}$$

Apply Ohm’s law to $R_2$

$$V_{\text{out}} = IR_2.$$  \hspace{1cm} (4)

Combine Equation (3) and Equation (4) to eliminate $I$

$$V_{\text{out}} = V_{\text{in}} \frac{R_2}{R_1 + R_2}.$$  \hspace{1cm} (5)

Equation (5) is the commonly used equation for the voltage output of a voltage divider. It applies to the case where $I_{\text{load}} = 0$. Analysis in Section 2.3 shows that this simple formula is adequate in most cases.
2.2 Case of Finite Load Resistance: $I_{\text{load}} \neq 0$

A voltage divider (or potentiometer) can still operate in situations where the load current is not negligible. However, when $I_{\text{load}}$ is not small, Equation (5) is not an accurate prediction of $V_{\text{out}}$. Figure 4 shows the circuit diagram for a voltage divider with non-negligible $I_{\text{load}}$. $R_3$ is the load resistance.

The effective resistance of the circuit in Figure 4 is

$$R_{\text{eff}} = R_1 + \frac{R_2 R_3}{R_2 + R_3}$$  \hspace{1cm} (6)

and the total current is

$$I = \frac{V_{\text{in}}}{R_{\text{eff}}}.$$  \hspace{1cm} (7)

Kirchhoff’s voltage law requires

$$V_{\text{in}} = V_1 + V_{\text{out}}$$

where $V_1$ is the voltage drop across $R_1$ and $V_{\text{out}}$ is the voltage drop across the parallel combination of $R_2$ and $R_3$. Solve the preceding equation for $V_{\text{out}}$ to get

$$V_{\text{out}} = V_{\text{in}} - V_1.$$  \hspace{1cm} (8)

Apply Ohm’s law to $R_1$ to obtain an equation for $V_1$

$$V_1 = I R_1.$$  \hspace{1cm} (9)

Substitute Equation (7) for $I$ into the preceding equation to find a relationship between $V_1$ and $V_{\text{in}}$

$$V_1 = \frac{V_{\text{in}}}{R_{\text{eff}}} R_1 = \frac{V_{\text{in}} R_1}{R_{\text{eff}}},$$ \hspace{1cm} (10)

Substitute this expression for $V_1$ into Equation (8) to get

$$V_{\text{out}} = V_{\text{in}} - V_{\text{in}} \frac{R_1}{R_{\text{eff}}} = V_{\text{in}} \left[ 1 - \frac{R_1}{R_{\text{eff}}} \right]$$  \hspace{1cm} (11)

Substitute the formula for $R_{\text{eff}}$ from Equation (6) and use algebra to simplify the resulting equation to obtain

$$V_{\text{out}} = V_{\text{in}} \frac{R_2}{R_1 \left( \frac{R_2}{R_3} + 1 \right) + R_2}.$$  \hspace{1cm} (12)

Refer to the Appendix for the intermediate algebraic steps. Equation (12) is the formula for the voltage divider when the load current is not negligible. Remember that $R_3$ is the resistance of the load, which is a characteristic of the system to which the voltage divider is attached, not a part of the voltage divider itself. When $R_3 \to \infty$, the ratio $R_2/R_3 \to 0$ and Equation (12) simplifies to Equation (5).
2.3 Comparing the Infinite and Finite Load Resistance Cases

The analysis in Section 2.1 and Section 2.2 provide two models for the voltage output of a voltage divider. The case of infinite load resistance in Section 2.1 gives a simpler formula obtained by ignoring the load resistance altogether. The case of finite load resistance in Section 2.2 is more general and more accurate, especially when \( R_3 \) is not that much larger than \( R_2 \). This begs the question: to what degree and in what circumstances does a finite \( R_3 \) matter? We will answer that question with two different models.

First, consider the case where the load resistance is not infinite and when the two resistors in the voltage divider are equal, i.e., \( R_1 = R_2 \). Begin by rearranging Equation (11) slightly,

\[
\frac{V_{out}}{V_{in}} = \frac{R_2}{R_1 \left( \frac{R_2}{R_3} + 1 \right) + R_2} \tag{13}
\]

Now, set \( R_1 = R_2 \) and simplify

\[
\frac{V_{out}}{V_{in}} = \frac{1}{\left( \frac{R_2}{R_3} + 1 \right) + 1} = \frac{1}{\frac{R_2}{R_3} + 2} \quad \text{Special case: } R_1 = R_2 \tag{14}
\]

Remember that Equation (14) only applies when \( R_1 = R_2 \). Table 1 shows an example of applying Equation (14) when \( R_1 = R_2 = 10 \text{k}\Omega \) for a sequence of decreasing \( R_3 \) values. When \( R_3 \) is large, e.g., for \( R_3/R_2 = 1000 \) or \( R_3/R_2 = 100 \) in Table 1, the value of \( V_{out}/V_{in} \) is very close to value of 0.5 that is obtained by ignoring \( R_3 \) for the case of \( R_1 = R_2 \).

Now, consider a more general analysis of the role of \( R_3 \) without the restriction that \( R_1 = R_2 \). Figure 5 shows a potentiometer circuit with the output connected across a load resistor \( R_3 \). The values of \( R_1 \) and \( R_2 \) are adjusted by changing the position of the potentiometer wiper. For convenience, let \( R_t \) be the total resistance of the potentiometer

\[
R_t = R_1 + R_2 \tag{15}
\]

and let \( \alpha \) be the fractional position of the potentiometer wiper

\[
\alpha = \frac{R_2}{R_t}. \tag{16}
\]

<table>
<thead>
<tr>
<th>( R_3 ) (( \Omega ))</th>
<th>( R_3/R_2 )</th>
<th>( V_{out}/V_{in} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>10 M( \Omega )</td>
<td>1000</td>
<td>0.4998</td>
</tr>
<tr>
<td>1 M( \Omega )</td>
<td>100</td>
<td>0.498</td>
</tr>
<tr>
<td>100 k( \Omega )</td>
<td>10</td>
<td>0.476</td>
</tr>
<tr>
<td>10 k( \Omega )</td>
<td>1</td>
<td>0.333</td>
</tr>
</tbody>
</table>

Figure 5: Potentiometer circuit with a non-zero load resistor, \( R_3 \).
Note that when $R_3 = \infty$, $V_{\text{out}}/V_{\text{in}} = \alpha$. Substituting Equation (16) into Equation (13) and simplifying gives

$$\frac{V_{\text{out}}}{V_{\text{in}}} = \frac{\alpha R_t}{(1 - \alpha)R_t\left(\frac{\alpha R_t}{R_3} + 1\right) + \alpha R_t}$$

$$= \frac{\alpha}{(1 - \alpha)\left(\frac{\alpha R_t}{R_3} + 1\right) + \alpha}$$

(17)

The preceding equation shows that the potentiometer output is determined by two parameters: the position of the wiper, $\alpha$, and the ratio of the total potentiometer resistance to the load resistance, $R_t/R_3$. Usually we consider the case where $R_t/R_3 < 1$, i.e., where $R_3$ is large.

Figure 6 is a plot of Equation (17), with $\alpha$ on the horizontal axis and $R_3/R_t$ as the parameter. The solid line corresponds to $R_3 = \infty$, i.e., the simple potentiometer model where the load resistance is so large that it is not important. For a given $\alpha$, points A and B shows the range of $V_{\text{out}}/V_{\text{in}}$ when $R_3$ varies from $\infty$ to $R_t$. For a given, desired $V_{\text{out}}/V_{\text{in}}$, the points A and C show that $\alpha$ needs to change (the wiper needs to be moved) when $R_3$ is not large enough to be neglected.

The analysis in this section shows that the load resistance is not a big obstacle to using a potentiometer. The computations summarized in Table 1 demonstrate that with $R_3/R_2 > 100$, the effect of the load resistance on the output voltage is negligible. Figure 6 shows that even when $R_3/R_t = 1$, the effect of the output resistor can be compensated by making a modest adjustment to the potentiometer. We conclude that the simpler potentiometer formula, Equation (5), can be used for most engineering design calculations.

Figure 6: Influence of non-zero load resistor, $R_3$, on the output of the potentiometer circuit.
3 Application: Using a 9V Battery to Supply 5V Power

Consider the circuit in Figure 7 in which a potentiometer is connected to a nine volt battery. By adjustment of the potentiometer, the circuit in Figure 7 allows a 9V battery to supply a voltage between 0 and 9V. It is not an efficient way to control the power voltage since power is dissipated in $R_1$ with no useful gain. However, this is one simple situation where a potentiometer could be used.

Suppose you wanted to create a 5V power supply using the idea from Figure 7, but you wanted to use fixed resistors instead of a potentiometer. Neglecting the current to the load resistor, what values of $R_1$ and $R_2$ would give $V_{out} = 5V$ if $V_{in} = 9V$?

We begin the analysis by rearranging Equation (5). Solve for $R_1/R_2$:

\[
\frac{V_{out}}{V_{in}} = \frac{R_2}{R_1 + R_2} \implies \frac{V_{out}}{V_{in}} = \frac{1}{\frac{R_1}{R_2} + 1} \implies \frac{R_1}{R_2} + 1 = \frac{V_{in}}{V_{out}}
\]

\[
\therefore \frac{R_1}{R_2} = \frac{V_{in}}{V_{out}} - 1 \tag{18}
\]

Equation (18) allows us to find a ratio of resistors that will achieve a desired voltage output. For example, with $V_{in} = 9V$ and $V_{out} = 5V$ we obtain

\[
V_{in} = 9V, \quad V_{out} = 5V \implies \frac{R_1}{R_2} = \frac{9}{5} - 1 = \frac{4}{5}
\]

What values of standard resistance satisfy the constraint $\frac{R_1}{R_2} = \frac{4}{5}$? Table 2 gives some standard values of 5% accurate resistors. Inspecting combinations of resistors shows that three possible combinations of resistors satisfy the requirement: $R_1 = 12\Omega$ and $R_2 = 15\Omega$, or $R_1 = 16\Omega$ and $R_2 = 20\Omega$, or $R_1 = 24\Omega$ and $R_2 = 30\Omega$.

4 Summary

Table 3 shows the voltage divider schematics and output formulas for the two cases analyzed in Section 2. For most applications, we can safely use the simple formula obtained by assuming $R_3 \approx \infty$. 
Table 2: Standard resistor values for 5% accuracy. Resistors are mass produced at powers of 10 times the base values listed on the left. The values on the right are an example of the base values times 100. See [http://www.rfcafe.com/references/electrical/resistor-values.htm](http://www.rfcafe.com/references/electrical/resistor-values.htm).

<table>
<thead>
<tr>
<th>Base values</th>
<th>Base values ×100</th>
</tr>
</thead>
<tbody>
<tr>
<td>10, 11, 12, 13, 15, 16,</td>
<td>1000, 1100, 1200, 1300, 1500, 1600,</td>
</tr>
<tr>
<td>18, 20, 22, 24, 27, 30,</td>
<td>1800, 2000, 2200, 2400, 2700, 3000,</td>
</tr>
<tr>
<td>33, 36, 39, 43, 47, 51,</td>
<td>3300, 3600, 3900, 4300, 4700, 5100,</td>
</tr>
<tr>
<td>56, 62, 68, 75, 82, 91</td>
<td>5600, 6200, 6800, 7500, 8200, 9100</td>
</tr>
</tbody>
</table>

Table 3: Summary of voltage divider formulas. These formulas also apply when the two fixed resistors are replaced by potentiometer.

<table>
<thead>
<tr>
<th>Case</th>
<th>Schematic</th>
<th>Formula for $V_{\text{out}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$I_{\text{load}} = 0$</td>
<td><img src="#" alt="Schematic" /></td>
<td>$V_{\text{out}} = V_{\text{in}} \frac{R_2}{R_1 + R_2}$</td>
</tr>
<tr>
<td>$I_{\text{load}} \neq 0$</td>
<td><img src="#" alt="Schematic" /></td>
<td>$V_{\text{out}} = V_{\text{in}} \frac{R_2}{R_1 \left(\frac{R_2}{R_3} + 1\right) + R_2}$</td>
</tr>
</tbody>
</table>
5 Appendix A: Algebra to Simplify $1 - R_1/R_{\text{eff}}$

In this section we will obtain a simplified form of the expression $1 - R_1/R_{\text{eff}}$ that appears on the right hand side of Equation (11). Start by working with the ratio $R_1/R_{\text{eff}}$

$$\frac{R_1}{R_{\text{eff}}} = \frac{R_1}{R_1 + \frac{R_2R_3}{R_2 + R_3}} = \frac{R_1(R_2 + R_3)}{R_1(R_2 + R_3) + R_2R_3}$$

$$= \frac{R_1 \left( \frac{R_2}{R_3} + 1 \right)}{R_1 \left( \frac{R_2}{R_3} + 1 \right) + R_2}$$

Now, subtract the preceding result from 1:

$$1 - \frac{R_1}{R_{\text{eff}}} = 1 - \frac{R_1 \left( \frac{R_2}{R_3} + 1 \right)}{R_1 \left( \frac{R_2}{R_3} + 1 \right) + R_2} = \frac{R_1 \left( \frac{R_2}{R_3} + 1 \right) + R_2 - R_1 \left( \frac{R_2}{R_3} + 1 \right) - R_2}{R_1 \left( \frac{R_2}{R_3} + 1 \right) + R_2}$$

$$= \frac{R_2}{R_1 \left( \frac{R_2}{R_3} + 1 \right) + R_2}$$