1   Overview

This document provides example problems to help students in ME 120 review and practice their skills with algebra.

2   Evaluating Simple Expressions

Evaluate each of the following expressions

1. Evaluate $(y - 1)^2$ for $y = -6$.

2. Multiply 7/16 by 4/3 and present the result as a fraction.

3. Divide 3/4 by 1/2 and present the result as a fraction.

3   Solving Equations of One Variable

3.1   Analytical Solutions

In this section, a series of problems are presented that have a solution that can be obtained in the form $x = \ldots$, where x is the single unknown, and $\ldots$ is an algebraic expression involving numerical constants.

1. Solve $x - 15 = -2(2x - 5)$ for x

2. Solve $4(y - 5) = -2(y + 2)$ for y

3. Solve for x in $x = \dfrac{2}{1 + x}$.

4. Solve for x in $x = \dfrac{1}{1 + 2x}$.

5. Solve for x in $x = \dfrac{5}{4 + x}$.

3.2   Graphical Solutions

In this section, the algebraic problems from the Section 3.1 are solved graphically. The advantages of the graphical approach are

- The graphical interpretation strengthens our intuition about both the "shape" of the function as well as understanding whether the solution may encounter difficulties

- The graphical method generalizes to problems where an analytical solution does not exist.

- It is good practice to develop skills at plotting functions.

Problems:

1. Given $x = \dfrac{2}{1+x}$,

   a. Create a plot of $f_1(x) = x$ and $f_2(x) = 2/(1+x)$ on the same set of axes. Adjust the ranges of the x and $y = f(x)$ axes until you can find two points where $f_1(x) = f_2(x)$. Those points are the solutions to the given equation.

   b. Rearrange the equation to the form, $ax^2 + bx + c = 0$. Plot $y = ax^2 + bx + c$ for a range of x values that includes the two ponts where $y = 0$. Those points are the solutions to the original equation. The solutions in part (b) should match the solutions to part (a), and should be the same as the analytical solution.

2. Repeat the graphical analysis of Problem 1 for $x = \dfrac{1}{1+2x}$.

3. Repeat the graphical analysis of Problem 1 for $x = \dfrac{5}{4+x}$.

4 Finding the equation of a line

1. Find the slope and intercept for the line passing through a given pair of points. Since only two points are given and the points are different, there is a unique solution.

   Use the notation $y = ax + b$ for the equation of the line, i.e.,. a is the slope and b is the intercept. Use hand calculations and show your work. Do not use a calculator.

   Note: Some of the point pairs have obvious answers (or will have obvious answers after you do some algebra). Instead of just writing down an obvious answer, use the given problems as algebra practice by working out the details and to show that your calculations are consistent with the obvious answers.

   a. $(-5, 1)$, $(6, 1)$

   b. $(-5, -4)$, $(-3, 5)$

   c. $(-4, -4)$, $(5, 5)$

   d. $(-3, 4)$, $(5, -2)$

   e. $(2, 2)$, $(6, 1)$

   f. (x_1, y_1), (x_2, y_2)

2. Use the solution to item 1f of the previous problem to create an Excel spreadsheet to check your answers. The following image shows one way to lay out the spreadsheet.

	A	B	C	D	E	F
1	Slope and Intercept of lines based on end-points					
2						
3	x1	y1	x2	y2	slope	intercept
4	-5	1	6	1		
5	-5	-4	-3	5		
6	-4	-4	5	5		
7	-3	4	5	-2		
8	2	2	6	1		
9						

5   Evaluating Expressions with Summation Notation

Evaluate the following expressions. The result is a single numerical value.

1. $\displaystyle\sum_{i=1}^{5} i$.

2. $\displaystyle\sum_{i=1}^{5} (i-1)$.

3. $\displaystyle\sum_{i=1}^{5} (i-1)^2$.

4. $\displaystyle\sum_{i=1}^{5} (1-i)^2$.

5. $\displaystyle\sum_{i=1}^{5} \frac{1}{(i-1)^2}$.

Evaluate the following expressions that use summation notation and a given data set. The limits of the sum are implied by the length of the available data set. The result is a single numerical value.

1. Evaluate $\sum x_i$ for $x_i = 1, 5, 7, 11, 13$.

2. Evaluate $\sum y_i$ for $y_i = 15, 11, 7, 3, -2$.

3. Evaluate $\sum x_i^2$ for $x_i = 1, 5, 7, 11, 13$.

4. Evaluate $\sum x_i y_i$ for $(x_i, y_i) = (1, 15), (5, 11), (7, 7), (11, 3), (13, -2)$.

5. Evaluate $\sum x_i \sum y_i$ for $(x_i, y_i) = (1, 15), (5, 11), (7, 7), (11, 3), (13, -2)$.

6 Word Problems

From `ipracticemath.com`

1. Find three consecutive numbers that add up to 216.

2. Five friends go to a movie. One of them uses a debit card to pay $45 for all of the tickets and $25 for all of the popcorn. How much does each of the four people without the debit card own the person who paid?

3. A rectangle has an area of $72 \, \text{cm}^2$, and is twice as long as it is wide? What are the dimensions of the rectangle?

7 Word Problems with Symbols

When solving engineering problems, it is often beneficial to create a general purpose formula instead of working with specific numerical values. The solutions to the following word problems are expressed in symbols, and therefore can be applied to reasonable choices of numerical values.

1. The schematic in Figure 1 depicts a swimming pool of length L and width W. A concrete sidewalk of width h surrounds the four sides of the pool. Suppose that the concrete is poured to a uniform depth b. What volume of concrete do you need to order from the Acme Sand and Gravel Company to make the sidewalk?
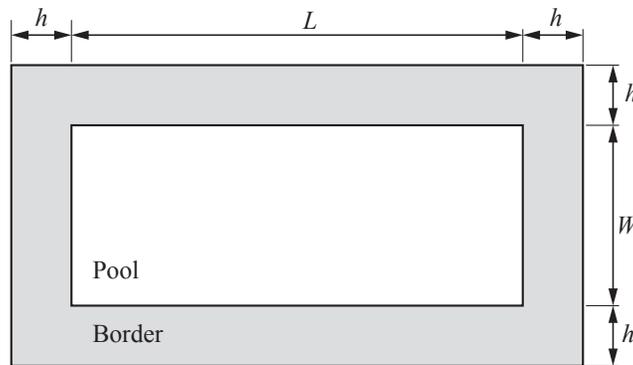

Figure 1: A sidewalk border around a rectangular swimming pool.