 gerry@pdx.edu

1 Introduction

A large body of engineering science provides us with theories and formulas for modeling the physical world. Correct use of those formulas requires an understanding of physics. It also requires attention to algebraic and numerical details. An important aspect of attention to detail is the correct use of units for physical quantities appearing in engineering formulas.

Engineers should always carry units with all formulas. Carrying and checking units is one way to maintain consistency in the use of formulas. At the very least, checking units helps us to prevent silly errors. However, having consistent units is not a guarantee that the formula is correct. In other words, checking for consistency of units is a one-sided kind of test: unit checks can find errors, but unit checks cannot prove correctness.

1.1 Learning Objectives

After studying these notes you should be able to

- Explain the difference between dimensions and units.
- Write the dimensions for any formula involving physical quantities, e.g., Newton's law of motion.
- Write the dimensions of the derived quantities in Table 2.
- Explain the principle of dimensional homogeneity
- Explain how the additive rule for units is more restrictive than the principle of dimensional homogeneity.
- Use the multiplication rule to convert units.
- Correctly distinguish units of physical quantities that may have the same numerical value, e.g., the density and volume of water in CGS units.

2 Dimensions of Physical Quantities

Before discussing units, we introduce the more fundamental concept of a dimension. In this context, a dimension is more general than the spatial coordinates associated with the three-dimensional space that we enhabit. When discussing units, a *dimension* is a measure of a physical quantity that does not have a numerical value associated with it[1].

For example, *length* and *area* are dimensions associated with the size of an object. We can refer to an object's length without giving a value for that length. We can say that two objects have the same length without providing

[1]NIST refers to *base quantities* instead of dimensions. See e.g., `https://physics.nist.gov/cuu/Units/units.html`.

Table 1: Basic dimensions. The symbols for basic dimensions are capital letters in non-italic shape.

Dimension	Symbol	Description
Length	L	Distance between to points; a measure of the geometric size of a an object.
Mass	M	The amount of matter; mass does not change when an object is moved to another gravitational field.
Time	T	temporal duration

the numerical values for those lengths. We can even show that they have the same lengths, for example, by placing then next to each other, without giving a value for that length.

To summarize:

- A dimension is a *type* of quantitative measure without reference to a system of units or to a numerical value in any one system of units.
- Units are quantitative measures established in a standardized system with respect to a precise, absolute reference.

Length is a dimension. A meter is a unit of length, defined in a system of units, and having a precisely defined value in that system.

2.1 Basic Dimensions

Basic dimensions are indivisible in the sense that they cannot be defined by combinations of other dimensions. Table 1 lists three basic dimensions used in engineering mechanics. We use the symbols L, M, and T as shorthand for these dimensions. It can be confusing because those same symbols are also used as engineering variables. With practice you will be to recognize when L (non-italic) is used to represent the length dimension and when L (in italic) is a variable that defines a specific length in an engineering problem. To further clarify meaning as we manipulate dimensions, we use square brackets around dimensions in algebraic expressions.

2.2 Derived Dimensions

The dimensions of force can be deduced from Newton's law of motion

$$F = ma \tag{1}$$

where F is a force acting on mass m and causing it to accelerate at rate a. Dimensions obey standard algebraic rules so

$$\text{dimension of force} = (\text{dimension of mass}) \times (\text{dimension of acceleration})$$

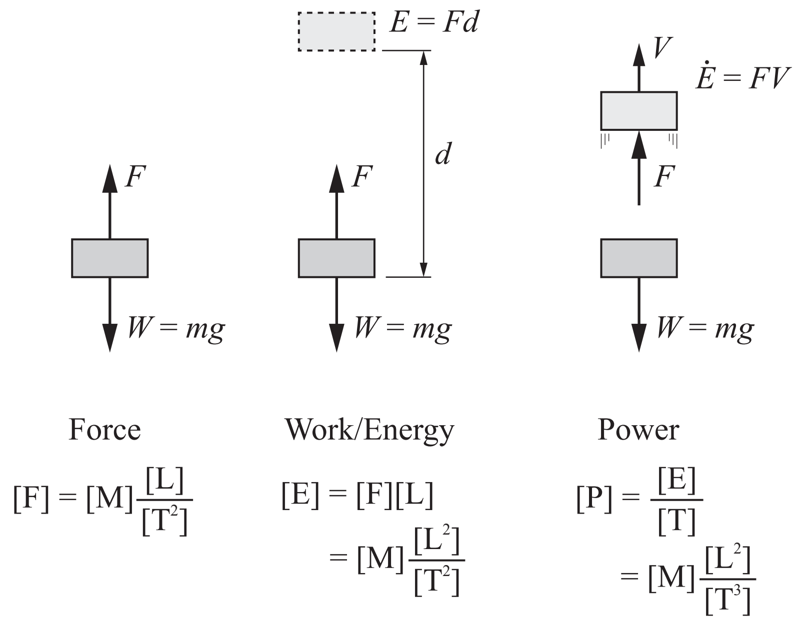


Figure 1: The derived dimensions of force, work and power can be obtained by considering the force required to hold, lift and move an object.

Mass has the basic dimension, M. The dimension of acceleration can be deduced from calculus

$$\text{dimension of acceleration} = \text{dimension of } \frac{d^2x}{dt^2} = \frac{[\mathrm{L}]}{[\mathrm{T}^2]}$$

To be extra clear, we use square brackets to indicate that the symbols L, M, and T are interpreted as dimensions. In the preceding expression you can read [L] as "the dimensions of length" and [T^2] as "the dimensions of time, squared". Therefore,

$$\text{dimension of force} = [\mathrm{M}] \left[\frac{\mathrm{L}}{\mathrm{T}^2} \right]$$

or,

$$[\mathrm{F}] = [\mathrm{M}] \left[\frac{\mathrm{L}}{\mathrm{T}^2} \right] \tag{2}$$

where [F] is understood to mean "dimensions of force".

The preceding development shows that the dimension of force are constructed as a combination of other dimensions. We call this type of dimension a *derived dimension* to distinguish it from the basic dimensions, M, L, and T, that cannot be expressed in terms of other dimensions.

With the dimensions of force established, we can derive the dimensions of other terms that involve force. Figure 1 depicts three physical concepts related to force acting on an object in a gravity field:

- Force necessary to hold an object of mass m in static equilibrium;
- Work is required to lift that object a distance d;

- Power is the rate of energy expenditure required to move that object with velocity V.

Figure 1 also shows the derived units for force, work and power.

Example 2.1 Dimensions of Momentum

Momentum is defined as the product of mass and velocity. Therefore

$$[\text{momentum}] = [\mathrm{M}]\frac{[\mathrm{L}]}{[\mathrm{T}]} = \frac{[\mathrm{M}]\,[\mathrm{L}]}{[\mathrm{T}]}$$

$\square$

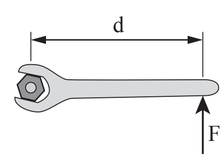
Example 2.2 Dimensions of Pressure

Pressure is defined as a force per unit area. Therefore

$$[\text{pressure}] = \frac{[\mathrm{F}]}{[\text{area}]} = \frac{[\mathrm{F}]}{[\mathrm{L}^2]} = \frac{1}{[\mathrm{L}^2]}[\mathrm{M}]\frac{[\mathrm{L}]}{[\mathrm{T}^2]} = \frac{[\mathrm{M}]}{[\mathrm{L}][\mathrm{T}^2]}$$

$\square$

Example 2.3 Dimensions of Torque



Torque is a force acting with a lever arm to create a tendency of an object to rotate. For example, as depicted in the sketch to the right, applying force perpendicular to the axis of a wrench will cause a twisting action on a nut. The torque on the nut is Fd, where F is the force and d is the distance between the line of action of the force and the center of pivot for the nut. The dimensions of torque are

$$[\text{torque}] = [\mathrm{F}]\,[\mathrm{L}]$$

$\square$

Table 2 lists some derived dimensions commonly used in Newtonian mechanics. In practice, we write [F] for units of force in engineering formulas and then substitute the basic dimensions of $[\mathrm{M}][\mathrm{L}]/[\mathrm{T}^2]$ when we need to algebraically simplify or check dimensions in a formula. Later we introduce the *units* of force in the SI system.

Table 2: Derived dimensions associated with basic Newtonian mechanics.

Area	$[\mathrm{L}^2]$	Force	$[\mathrm{F}] = [\mathrm{M}] \times \dfrac{[\mathrm{L}]}{[\mathrm{T}^2]} = \dfrac{[\mathrm{M}]\,[\mathrm{L}]}{[\mathrm{T}^2]}$
Volume	$[\mathrm{L}^3]$	Work	$[\mathrm{F}] \times [\mathrm{L}] = \dfrac{[\mathrm{M}]\,[\mathrm{L}^2]}{[\mathrm{T}^2]}$
Velocity	$\dfrac{[\mathrm{L}]}{[\mathrm{T}]}$	Power	$[\mathrm{F}] \times \dfrac{[\mathrm{L}]}{[\mathrm{T}]} = \dfrac{[\mathrm{M}]\,[\mathrm{L}^2]}{[\mathrm{T}^3]}$
Acceleration	$\dfrac{[\mathrm{L}]}{[\mathrm{T}^2]}$	Torque	$[\mathrm{F}] \times [\mathrm{L}] = \dfrac{[\mathrm{M}]\,[\mathrm{L}^2]}{[\mathrm{T}^2]}$

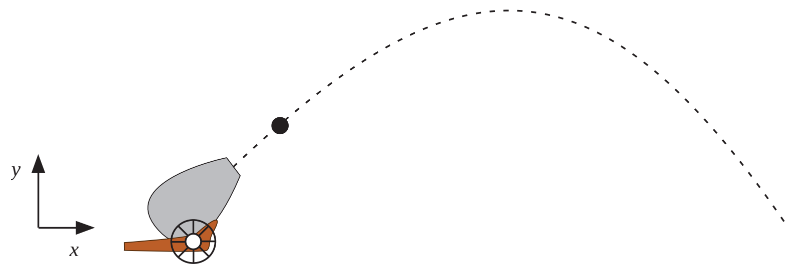


Figure 2: Ballistic trajectory of a cannon ball.

2.3 Equations Must Be Dimensionally Homogeneous

The principle of dimensionally homogeneity requires that each additive term in an equation must have the same dimensions. For example, we cannot add a length to a mass. Figure 2 shows a schematic representation of the flight of a cannon ball. The general equations for the trajectory of the ball are

$$\begin{array}{ccccccc} x(t) & = & x_0 & + & V_{x,0}t & + & a_x t^2 \\ y(t) & = & y_0 & + & V_{y,0}t & + & a_y t^2 \\ & & & & & & \\ [\mathrm{L}] & & [\mathrm{L}] & & [\mathrm{L}] & & [\mathrm{L}] \end{array} \tag{3}$$

where x and y are the position coordinates, (x_0, y_0) is the initial position of the ball, $(V_{x,0}, V_{y,0})$ is the initial velocity of the ball, (a_x, a_y) are the acceleration of the ball. Each of the terms in Equations (3) must have dimensions of length.

2.4 Dimensions of Angles and π

For engineering analysis, angles should be measured in radians, not degrees. A radian is defined as the ratio of arc length to radius, as show in Figure 3.

$$\theta = \frac{s}{r}$$

Because the angle is a ratio of lengths, it is dimensionless, meaning that it has no dimension. However, although angles have dimensions, angles do have a unit, which is the radian.

The pure number π is defined as the ratio of a the circumference of a circle to its diameter. Since circumference and diameter are both lengths, π is dimensionless.

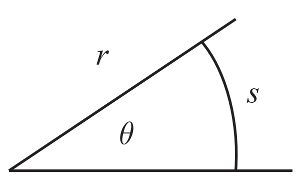


Figure 3: The angle θ is defined as the ratio s/r.

Table 3: Units of basic dimensions in the SI and British gravitational systems of units.

Dimension	SI Unit	English engineering unit
Length	meter (m)	foot (ft)
Mass	kilogram (kg)	$\mathrm{lb_m}$
Time	second (s)	second (s)
Temperature	kelvin (K)	degree Fahrenheit (°F)
Electric current	Ampere (A)	Ampere (A)
Amount of substance	Mole (mol)	Mole (mol)
Luminous intensity	Candela (cd)	Candela (cd)

3 Units

Units are standardized measures associated with dimensions. Units have numerical values in carefully defined and standardized *systems of units*.

A *systems of units* is a set of units with conventional names and precise and definitions. The most common systems of units used in engineering are SI (*Systèm internationale (d'unités)* or International System of Units) and English Engineering units. Table 3 list the seven base units (or dimension, in the terminology used in previous sections) listed by the National Institute of Standards and Technology[2], along with the SI and English Engineering units for those dimensions.

For modern engineering and global commerce, the SI system is preferred. All major industrial countries except the United States use SI[3]. However, due to political inertia and a large legacy of installed manufacturing equipment, English Engineering units are still widely used in the United States[4].

3.1 Addition Rule for Equations with Units

The principle of dimensional homogeneity extends to units. All terms in a sum must have the same units. We can call this the *addition rule*, which is just the logical requirement of applying dimensional homogenity to units. Consider, for example

$$A = B + C. \tag{4}$$

[2] `https://www.nist.gov/pml/weights-and-measures/si-units`

[3] According to Wikipedia, in 2018 the only countries not adopting SI units are the United States, Myanmar, Liberia, Palau, the Marshall Islands, the Federated States of Micronesia, and Samoa. Of those countries, Myanmar and Liberia use SI units practically, and Palau, Micronesia and the Marshall Islands are administered by the United States federal government. `https://en.wikipedia.org/wiki/Metrication_in_the_United_States`.

[4] NIST publication 1136a, `https://www.nist.gov/sites/default/files/documents/pml/wmd/metric/1136a.pdf`, provides a concise history of attempts to adopt the metric in the United States. For example, Congress passed the *Metric Conversion Act* in 1975. Additional support was provided by the *Omnibus Trade and Competetiveness Act* of 1988, which required federal agencies to us the SI system by 1992. More details are provided in NIST Special Publication 811, *The NIST Guide for the use of the International System of Units*, `https://www.nist.gov/physical-measurement-laboratory/nist-guide-si-preface`

For Equation (4) to be valid, the units of A must match the units of B, which must match the units of C. Of course, the dimensions of A must match the dimensions of B, which must match the dimensions of C. However, *the requirement of equality of units is more strict than the equality of dimensions.*

Example 3.1 How tall is Jane?

Using English Engineering units, Jane says she is 5 feet, 10 inches tall. How tall is Jane in feet? How tall is Jane in inches?

Let H be the symbol used for Jane's height. The addition rule *prevents* us from writing H as $5\,\mathrm{ft} + 10\,\mathrm{inch}$, or $60\,\mathrm{inch} + 0.8333\,\mathrm{ft}$. To correctly add the two height quantities (5 ft and 10 inch), the two quantities have to have the same units, either ft or inch. Therefore, we can correctly write the *equations*

$$
\begin{aligned}
H &= 60\,\mathrm{inch} + 10\,\mathrm{inch} = 70\,\mathrm{inch} \\
&\text{or} \\
H &= 5\,\mathrm{ft} + 0.8333\,\mathrm{ft} = 5.8333\,\mathrm{ft}.
\end{aligned}
$$

To convert feet to inches or inches to feet, use the multiplication rule, which is discussed next.

$\square$

Multiplication Rule for Converting Units

A basic rule of algebrais that we can always multiply any term in an equation by one, without changing the correctness of the equation. For example, we can multiply any term in Equation (4) by 1. For example

$$A = B + C \times 1 \tag{5}$$

In unit conversions, we multiply by a *ratio* of two equal quantities, which is equivalent to multiplying by 1. For example, the exact relationship between inches and centimeters is

$$1 \text{ inch} = 2.54 \text{ cm} \tag{6}$$

Dividing through by the right hand side gives

$$\frac{1 \text{ inch}}{2.54 \text{ cm}} = 1 \tag{7}$$

Or, we can divide Equation (6) by the left hand side to get.

$$1 = \frac{2.54 \text{ cm}}{1 \text{ inch}} \tag{8}$$

Equation (7) and Equation (8) are examples of conversion factors we use in converting units.

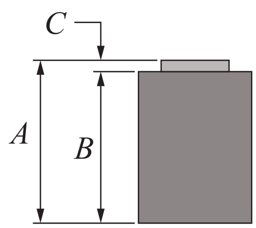


Figure 4: A small box of height C stacked onto a larger box of height B.

Example 3.2 How tall is a stack of boxes?

Figure 4 represents a stack of two boxes. Suppose the height of the bottom box is $B = 2\,\text{ft}$ and the heigh of the top box is $C = 1.5\,\text{inch}$. The total height is $A = B + C$, which has consistent dimensions because B and C both have dimensions of length. However, we cannot add 2 ft to 1.5 inch because those two quantities have different units. Although this example is trivial, it is surprising how often inexperienced (or sloppy) engineers make errors by adding quantities with unequal units.

One way to correctly add the height of the two objects is to convert the height of the small box to feet to get

$$A = 2\,\text{ft} + 1.5\,\text{inch} \times \frac{1\,\text{ft}}{12\,\text{inch}} = 2\,\text{ft} + 0.125\,\text{ft} = 2.125\,\text{ft}$$

Alternatively, we could convert the height of the larger box to inches

$$A = 2\,\text{ft} \times \frac{12\,\text{inch}}{1\,\text{ft}} + 1.5\,\text{inch} = 24\,\text{inch} + 1.5\,\text{inch} = 25.5\,\text{inch}$$

Of course, being careful engineers, we check that these two answers are equivalent

$$A = 2.125\,\text{ft} \times \frac{12\,\text{inch}}{1\,\text{ft}} = 25.5\,\text{inch}$$

which shows that the two methods of computing the height are equivalent.

$\square$

Example 3.3 How tall is Jane?

From Example 3.1 above, Jane's height in feet is

$$H = 5\,\text{ft} + 10\,\text{inch}\,\frac{1\,\text{ft}}{12\,\text{inch}} = 5\,\text{ft} + 0.833\,\text{ft} = 5.833\,\text{ft}$$

Alternatively, her height in inches is

$$H = 5\,\text{ft}\frac{12\,\text{inch}}{1\,\text{ft}} + 10\,\text{inch} = 60\,\text{inch} + 10\,\text{inch} = 70\,\text{inch}$$

To convert inches to meters, multiply by two conversion factors

$$H = 70\,\text{inch}\,\frac{2.54\,\text{cm}}{\text{inch}}\,\frac{1\,\text{m}}{100\,\text{cm}} = 1.78\,\text{m}$$

Note that there is no inconsistency in using H as the symbol for height in the three different units. $\square$

Table 4: SI Units of force, energy and power.

Physical Quantity	Symbol	Definition
Force	N	$1\,\text{newton} = 1\,\text{kg}\,\dfrac{1\,\text{m}}{1\,\text{s}^2}$
Energy	J	$1\,\text{joule} = 1\,\text{N}\,\text{m}$
Power	W	$1\,\text{watt} = \dfrac{1\,\text{J}}{1\,\text{s}}$

Example 3.4 How big is the floor area of a room?

A room with a floor dimension of 12 ft by 16 ft has an area of $192\,\text{ft}^2$. What is the floor area in m^2?

One way to solve this problem is to convert each of linear dimensions and multiply them together.

$$12\,\text{ft}\,\frac{12\,\text{in}}{\text{ft}}\,\frac{2.54\,\text{cm}}{\text{in}}\,\frac{1\,\text{m}}{100\,\text{cm}} = 3.66\,\text{m}$$

$$16\,\text{ft}\,\frac{12\,\text{in}}{\text{ft}}\,\frac{2.54\,\text{cm}}{\text{in}}\,\frac{1\,\text{m}}{100\,\text{cm}} = 4.88\,\text{m}$$

Now we can compute the area in m^2

$$A = (3.66\,\text{m})(4.88\,\text{m}) = 17.86\,\text{m}^2.$$

An alternative approach is to work with the floor area directly. That technique would be necessary, for example, if you were given the floor area without the individual length and width of the room.

$$A = 192\,\text{ft}^2\left[\frac{12\,\text{in}}{\text{ft}}\right]^2\left[\frac{2.54\,\text{cm}}{\text{in}}\right]^2\left[\frac{1\,\text{m}}{100\,\text{cm}}\right]^2 = 17.84\,\text{m}^2$$

The slight difference in values for A is due to roundoff errors in convert 16 ft to m.

□

3.2 Derived Units of Force, Energy and Power

Table 2 gives the basic dimensions for force, work/energy and power. In SI and Engineering Engineering systems of units, there are additional names associated with those quantities. In SI the unit of force is newton; the unit of energy is joule, and the unit of power is watt. Table 4 gives the definitions of those derived units in terms of force units. Note that when we write out the names of the units, the first letter is not capitalized, even though the unit is named after a famous person. In contrast, the one-letter symbol is always capitalized.

4 Potential Pitfalls

As with any technical field, there are many ways to make errors in computations. Here we point out two potential pitfalls when working with units.

4.1 Quantities with Equal Values and Unequal Units

Before concluding a discussion of units, we connect the concept of units with physical relationships *between* distinctly different, but related, quantities. We will use a simple example to demonstrate the importance of carrying units to avoid confusing two related physical quantities that are not the same, even when those physical quantities have the same numerical value.

Consider the relationship between mass, m, density, ρ, and volume, $\mathcal{V}$, which is

$$m = \rho \mathcal{V}. \tag{9}$$

Density is an intensive property of a material. In this context *intensive* means not dependent on size. For example, at the same temperature, the density of water is the same whether the sample is the size of a test tube or the size of a drinking glass or the size of a swimming pool. In contrast, both the mass, m, and volume, $\mathcal{V}$, are extensive properties. The mass (or volume) of water in a test tube is not the same as the mass (or volume) of water in a full glass of water or the mass (or volume) of water in a swimming pool.

Equation (9) expresses a relationship between volume and mass. If we know the volume of a sample and the density of the material in that sample, we can use Equation (9) to compute the mass in the sample.

Consider a container holding $250\,\mathrm{cm}^3$ of water. The mass of water in the container is

$$m = \left(1\,\frac{\mathrm{gm}}{\mathrm{cm}^3}\right)\left(250\,\mathrm{cm}^3\right) = 250\,\mathrm{gm}$$

In this case, the mass of the water has the same numerical value (250), in units of gm, as the volume of the water (250), in units of cm^3. However, although m and $\mathcal{V}$ have the same numerical value, m and $\mathcal{V}$ are not the same physical quantity.

4.2 Relative Temperature Scales

An exception to the multiplication rule from unit conversion is the changing temperature in units of °C to units of °F, and vice versa. The problem is that the Celcius and Fahrenheit are not absolute temperature units. Figure 5 is a graphical representation with the boiling and freezing points of water as the references. The diagram shows what we also know: 32 °F is the same temperature as 0 °C. The zero values of these temperature scales are not at the same temperature. Therefore, the relationship between these temperature units cannot be expressed as a simple ratio.

Consider an object at a stable temperature. Let T_F be the temperature of that object in °F, and let T_C be temperature of that object in °C. The conversion is

$$T_C = (T_F - 32)\frac{5}{9} \tag{10}$$

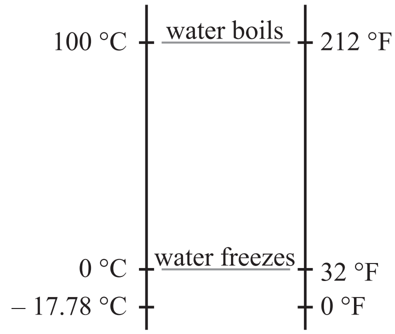


Figure 5: Celcius and Fahrenheit temperature scales are not absolute – their zero values do not occur at the same temperature.

or

$$T_F = \frac{5}{9} T_C + 32. \tag{11}$$

When temperature differences are concerned, the misalignment of zero degrees on the two temperature scales does not matter.

Consider the indoor-to-outdoor temperature difference depicted in Figure 6. Before we go further, double check that the indoor and outdoor temperature values in the sketch are converted correctly.

Let T_{in} and T_{out} be the indoor and outdoor temperatures, respectively. The temperature differences in the two systems of units are

$$\Delta T_C = T_{\text{out}} - T_{\text{in}} = 20 - 5 = 15\,^\circ\text{C} \tag{12}$$
$$\Delta T_F = T_{\text{out}} - T_{\text{in}} = 68 - 41 = 27\,^\circ\text{C} \tag{13}$$

A simple calculation shows that for this case $\Delta T_C = (5/9)\Delta T_F$. In general for any two temperatures T_1 and T_2

$$T_{1,F} - T_{2,F} = \left[\frac{9}{5} T_{1,C} + 32\right] - \left[\frac{9}{5} T_{2,C} + 32\right] = \frac{9}{5}\left(T_{1,C} - T_{2,C}\right)$$

because the offset of 32 °F cancels when temperature differences are converted.

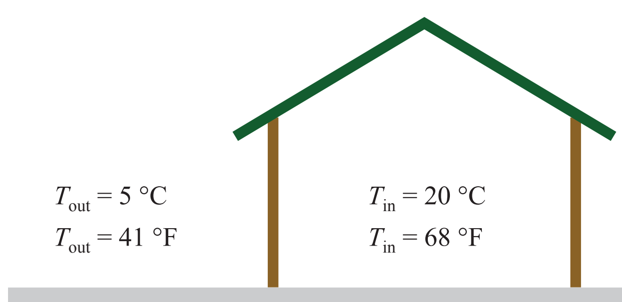


Figure 6: Indoor and outdoor temperature differences in °C and °F.

5 Summary

The important ideas from these notes are:

- A dimension is a type of quantitative measure without reference to units or specific numerical values. Basic dimensions are length, mass and time, which (in these notes) are given the symbols L, M and T.

- Units are standard measures associated with dimensions and that are defined with respect to reference quantities within a *system of units*. Basic units in the SI system are m (meter), kg (kilogram) and s (second). The corresponding units in the English Engineering system are ft (feet), $\mathrm{lb_m}$ (pound mass) and s (second).

- Formulas that describe physical phenomena, e.g., Newton's law of motion, must express a consistent relationship between dimensions. In the case of Newton's law, the dimensions of force are *defined*, by the dimensions of the formula
$$F = ma \quad \Longrightarrow \quad [\text{force}] = \frac{[\mathrm{M}]\,[\mathrm{L}]}{[\mathrm{T}^2]}$$
where [L] is read "dimensions of length", etc.

  The dimensions of derived quantities are expressed from the formulas that define those quantities, as summarized in Table 2.

- The principle of dimensional homogeneity requires that all terms that add to each other in an equation must have the same dimensions.

- A given dimension can be expressed with different units. For example, length can be expressed in meters, centimeters, feet or inches. When adding quantities the *units* must be the same. Therefore, since units are more specific than dimensions, the additive rule for units is more restrictive than the principle of dimensional homogeneity.

  Simple Rule: Addition can only involve quantities with the same units.

- Convert units by multiplication of ratios that are equal to one. For example, convert 15 inches to centimeters with the following,
$$2.54\,\mathrm{cm} = 1\,\mathrm{in} \quad \Longrightarrow \quad \frac{2.54\,\mathrm{cm}}{1\,\mathrm{in}} = 1$$
Therefore
$$15\,\mathrm{in} \times \frac{2.54\,\mathrm{cm}}{1\,\mathrm{in}} = 38.1\,\mathrm{cm}$$
A series of multiplication-by-one conversions can be combined. For example, to convert 3 miles to kilometers
$$3\,\mathrm{mile} \times \frac{5280\,\mathrm{ft}}{1\,\mathrm{mile}} \times \frac{12\,\mathrm{in}}{\mathrm{ft}} \times \frac{2.54\times10^{-2}\,\mathrm{m}}{\mathrm{in}} \times \frac{1\,\mathrm{km}}{1000\,\mathrm{m}} = 4.83\,\mathrm{km}$$

- Physical quantities with the same numerical value, but not the same units are not equal. This confusion often arises when performing computations involving the density of water in SI or CGS units. To one decimal place, the density of water is

$$\rho_w = 1\,\frac{1\text{gm}}{\text{cm}^3}$$

The mass of water in a $200\,\text{cm}^3$ is

$$m = \rho\mathcal{V} = \frac{1\text{gm}}{\text{cm}^3} \times 200\,\text{cm}^3 = 200\,\text{gm}$$

Although the mass ($200\,\text{gm}$) and volume ($200\,\text{cm}^3$) have the same numerical value, the mass and volume are two distinctly different physical quantities with different units.

- Temperature conversion from °C to °F is complicated because the two scales are zero at different temperatures. Either Equation (10) or Equation (11) is needed convert between °C and °F, unless the conversion is for temperature differences.

6 Practice Problems

Use the following problems to practice working with units. For best results (i.e., best practice), do not use an on-line unit conversion tool. Instead, write out the conversion factors as *multiply-by-one* terms. You may need to look up the factors that allow you to create the multiplicative factors.

1. What are the dimensions and SI units of each of the following expressions?

   a. $\dot{m} = \rho v A$, where ρ is a density, v is a velocity, and A is an area

   b. $\frac{1}{2}\rho v^2$, where ρ is density and v is velocity.

   c. $\dot{m}gh$, where $\dot{m}$ is a mass flow rate (kg/s), g is the acceleration of gravity, and h is a height.

   d. $\frac{1}{2}\dot{m}v^2$, where $\dot{m}$ is a mass flow rate (kg/s), and v is velocity

2. Use the multiplication rule to evaluate the following

   a. Convert $5\,\text{in}^2$ to cm^2.

   b. Convert 50 kph (kilometers per hour) to m/s.

   c. Convert 50 kph (kilometers per hour) to MPH (miles per hour).

   d. Convert $200\,\text{in}\,\text{lb}_\text{f}$ to N m.

   e. Convert 50 PSI to Pa.

3. If ω is an angular velocity in rad/s, r is a radius in m, and $v = r\omega$, what are the units of v? What is the physical significance of v?

4. Fill in the missing temperatures in the following table.

T (°C)	T (°F)
-30	
	-10
0	0
5	
	50
	80
30	

5. Use the multiplication rule to make the following currency conversions. Don't just use an on-line converter. Document the conversion with multiplication.

   a. 5 US dollars to Euros
   b. 10000 Chinese yuan to US dollars
   c. 20 US dollars to South African Rand

   How is the conversion of currencies different from the conversion of physical quantities such as converting meters to inches?