Introduction to DC Circuits Gerald Recktenwald
v 1.0: September 29, 2019 gerry@pdx.edu

1 Introduction

Engineers from all disciplines need to have working knowledge of basic electrical circuits. These notes introduce the following fundamental concepts.

1. Ohm's Law
2. Power dissipation
3. Resisters in series
4. Resistors in parallel

It is important to have more than just a knowledge of the vocabulary and concepts. Engineers need to be able to work with mathematical models of electrical circuits in order to predict the behavior of circuits, to selet and use sensors, to choose supporting components (resistors, capacitors, diodes, etc.) and to control motors, lights and other devices.

Electricity is present in all areas of technology. Electricity is a source of power for motors, actuators, lights, communication and computing systems. Electrical signals contain information and allow the transmission of information in computer networks, telecommunication systems, music and in the nervous systems of our bodies. For mechanical engineers, the key uses of electricity are for supplying power to motors, and to the transmission of and detection of signals used to measure and control the behavior of electromechanical systems.

There are two main ways of supplying electricity for power: alternating current or AC, and direct current or DC. AC power is available in the wall outlets of homes and businesses. AC is characterized by an smoothly oscillating voltage difference on two or three conducting wires. The oscillations of AC power are regulated at a few discrete frequencies: 50 Hz in Europe, Asia, Africa and Australia, and 60 Hz in the United States, Canada, Mexico and Brasil[1]. In contrast to the time-varying signal of AC power, DC power is a constant. A battery is a common source of DC power, as are the ubiquitous "brick" power supplies used for phones and other portable electric devices. In the remainder of this document, we will only be concerned with DC power.

2 Ohm's Law

Ohm's law is the quantitative relationship between electrical current and voltage when the current flows through a resistor or conductor. Ohm's law applies to individual resistive elements, e.g., lengths of wire and resistors, as well as networks of resistive elements.

Consider a wire or resistor with resistance R ohm (Ω) depicted in Figure 1. When a current of I amps (A) flows through the resistor, a voltage *drop* V_{AB} volts (V) appears across the terminals A and B.

$$V_{\mathrm{AB}} = IR$$

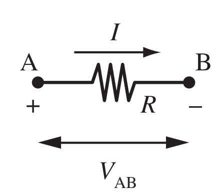


Figure 1: Ohm's law for a conductor.

We say that V_{AB} is a voltage drop because the voltage at A is higher than the voltage at B. The convention for

[1]The unit of frequency is hertz, abbreviated *Hz*, and named in honor of Heinrich Hertz, a German physicist who lived from 1857 to 1894.

electrical current is that the current flows from a higher voltage to a lower voltage. Therefore, the arrow associated with the current, I, in Figure 1 is pointed from A to B.

The double arrow $\leftrightarrow$ associated with V_{AB} in Figure 1 indicates the end points that correspond to V_{AB}. For the resistor in the diagram, there is no ambiguity about the physical end points that correspond to the + and − poles. Therefore, we can drop the designation of endpoints and simply write

$$V = IR \tag{1}$$

which is the basic form of Ohm's law.

The units of the terms in Ohm's Law *define* the relationship between the units of voltage, current and resistance.

$$V = IR \quad \Longrightarrow \quad 1\,\text{volt} = 1\,\text{amp} \times 1\,\text{ohm} \quad \text{or} \quad 1\,\text{V} = 1\,\text{A} \times 1\,\Omega$$

We can rearrange Ohm's law to obtain other useful formulas and see other relationships between units

$$I = \frac{V}{R} \quad \Longrightarrow \quad 1\,\text{amp} = \frac{1\,\text{volt}}{1\,\text{ohm}} \quad \text{or} \quad 1\,\text{A} = \frac{1\,\text{V}}{1\,\Omega}$$

$$R = \frac{V}{I} \quad \Longrightarrow \quad 1\,\text{ohm} = \frac{1\,\text{volt}}{1\,\text{amp}} \quad \text{or} \quad 1\,\Omega = \frac{1\,\text{V}}{1\,\text{A}}$$

3 Physical Definitions of Current, Voltage and Resistance

Ohm's law is a extremely important and practical relationship *between* voltage, current and resistance that determine the flow of electricity through a single conductor. In many engineering applications, we don't need to think about the atomic-level details that make electricity work the way it does. For example, to build a circuit that uses a sensor to measure temperature, we don't need to worry about protons, neutrons and the cloud of electrons bound to individual atoms. And most of the time we don't need to worry about the detailed mechanisms involved in the flow of electrons that are responsible for electrical current. However, if you want to understand how semiconductor devices, e.g., transistors, work, or the how the plasmas in fluorescent lights and lasers are created, you will need to dig deeper into the physics of electricity.

For our immediate purposes, we will only introduce the fundamental definitions of voltage, current and resistance as those quantities appear in Ohm's law. We do this to establish the relationships between the units of those quantities

Current is the flow of electrons, or equivalently, the flow of electrical charge. The unit of charge is *coulomb*, which is given the symbol C. An amp is defined as

$$1\,\text{A} = \frac{1\,\text{C}}{1\,\text{s}} = \frac{1\,\text{coulomb}}{1\,\text{second}} \tag{2}$$

Voltage is a measure of the work (or energy) necessary to separate opposite charges. The units of energy *joule* (J) and the units of charge are coulomb (C), and a volt is defined as

$$1\,\text{V} = \frac{1\,\text{J}}{1\,\text{C}} = \frac{1\,\text{joule}}{1\,\text{coulomb}} \tag{3}$$

Resistance is a macroscopic property of a fixed amount of material. Electrical conductors have an intrinsic property called the *resistivity*. In other words, whereas a fixed amount of material, say a length of wire, has a value of resistance, the material that makes up that wire has a resistivity that is independent of the amount.

The relationship between wire length, skinniness and intrinsic material properties are expressed by

$$R = \frac{\rho L}{A} \tag{4}$$

where ρ is the *resistivity*, L is the length of the wire, and A is the cross-sectional area of the wire. In most practical engineering situations we usually work with the macroscopic value of resistance, not the resistivity of the material. However, when comparing materials for a particular application, say in the design of a heating element, we use Equation (4).

Example 1 Electrical resistance of breadboard jumper wires

Wires used as jumpers on a breadboard are 22 AWG, where AWG is the abbreviation for American Wire Gage. A 22 AWG wire has a diameter of 0.655 mm (0.0253 inch). What is the resistance of a 22 AWG copper wire that is 1 m long?

Equation (4) predicts the resistance of a wire of known material (ρ), length L and cross-sectional area $A = (\pi/4)d^2$, where d is the diameter. From Wikipedia[2] the resistivity of copper is

$$\rho_{\mathrm{Cu}} = 1.68 \times 10^{-8}\ \Omega \cdot \mathrm{m}$$

The area of a 22 AWG wire is

$$A = \frac{\pi}{4} d^2 = \frac{\pi}{4}(0.644 \times 10^{-3}\mathrm{m})^2 = 3.257 \times 10^{-7}\mathrm{m}^2$$

Therefore, the resistance of a 1 m length of 22 AWG copper wire is

$$R = \frac{(1.68 \times 10^{-8}\ \Omega \cdot \mathrm{m})(1\,\mathrm{m})}{3.257 \times 10^{-7}\mathrm{m}^2} = 0.052\,\Omega.$$

Most jumper wires are a just a few centimeters long, which means the resistance of jumper wires is on the order of $(1/100)0.052\,\Omega \approx 0.0005\,\Omega$. Therefore, the resistance of typical a jumper wire is negligible in a circuit that includes $330\,\Omega$ and $10\,\mathrm{k}\Omega$ resistors.

□

4 Work, Energy and Power

We have already introduced the definitions of voltage, current and resistance. Next we turn to work, energy and power. In practical applications, we are concerned about the power consumed in resistive loads (e.g. heaters) or the energy capacity of batteries. We will focus on the power that is dissipated (i.e., lost) when current flows through individual resistors, or networks of resistors.

4.1 Definitions

Although power, work and energy have everyday meanings, we need to define these terms with a precision that is unambiguous and that allows us to perform quantitative calculations.

To introduce (or review) the concepts of work, energy and power, we will use the more familiar mechanical process of lifting a weight. When considering electrical circuits, work and energy are defined in terms of electrical field potential, which are more abstract. For both mechanical and electrical models, a key idea is it takes work to change the potential energy of an object. Furthermore, in the absence of any energy losses (e.g., for motion without friction) the amount of work done is equal to the change in potential energy.

[2] https://en.wikipedia.org/wiki/Electrical_resistivity_and_conductivity

Work and Energy

Work is a thermodynamic quantity with the same units as energy. Mechanical work can be defined as the energy necessary to raise a weight a given distance in a gravitational field. Figure 2 is a schematic representation of a mass m being raised through a distance d. The weight of the mass is $W = mg$ acts in the direction opposite of the motion. A force opposing gravity, and greater in magnitude[3] to the weight will cause the weight to move. The *minimum* amount of mechanical work to raise the weight is

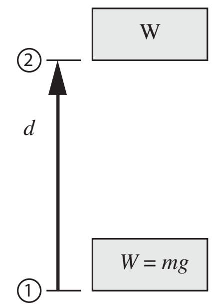


Figure 2: Raising a weight W through a distance d.

$$\text{Work} = \text{force} \times \text{distance} = (mg) \times d. \tag{5}$$

This is the minimum work because we are just balancing the weight, and an infinitesimally greater force would move the weight upward. In this scenario we are also ignoring any effect of friction losses such as sliding friction between the block and any fixed supports or air resistance. In the absence of friction, the (minimal) work done is equivalent to the change in potential energy

$$\text{Change in potential energy} = mgd. \tag{6}$$

The unit of work or energy is joule[4]. One joule is defined as a newton-meter

$$1\ \text{J} = 1\ \text{N} \times 1\ \text{m} \tag{7}$$

The unit of force is *Newton* and is defined from the units of Newton's law of motion, $F = ma$

$$1\,\text{N} = 1\ \text{kg} \times 1\ \frac{\text{m}}{\text{s}^2} \tag{8}$$

Work is done when a force acting on an object causes that object to move. The motion can be in any direction. For example, as depicted in Figure 3, pushing a block with a force F through a horizontal distance d does $F \times d$ units of work on the block. Unlike raising a weight (see Figure 2), pushing the block horizontally does not change the potential energy of the block. The work input is dissipated by friction between the block and the table surface. If the table was frictionless, or if the block was supported by rollers having zero friction, the force necessary to move the block would be theoretically zero, and hence no work would be done.

Power and Work

Power is the *rate* at which work is done. It is also the rate at which energy is expended. The SI unit of power is *watt*[5].

$$1\,\text{W} = \frac{1\,\text{J}}{1\,\text{s}} \tag{9}$$

[3] A force exactly equal to the weight would keep the block from moving downward, i.e., to hold it in static equilibrium.

[4] In honor of James Prescott Joule (1818–1889), who, among other achievements, built an elegant experiment to show the equivalents of mechanical work and thermal energy. Although the unit of energy is named after James Prescott Joule, when ”joule” is used as the unit of energy, it is not capitalized. The same is true for other units named after people who made important contributions to science, e.g. curie, newton, and watt.

[5] Named after James Watt, a Scottish engineer, who lived from 1736 to 1819.

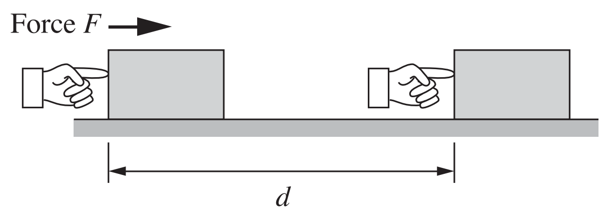


Figure 3: Pushing a block across a table does $F \times d$ units of work without any change in potential energy due to gravity. The work is dissipated by friction between the block and table surface.

4.2 Power Dissipation in a conductor

When electrical current flows through a simple conductor, like that in Figure 1, the power dissipation is

$$P = VI \tag{10}$$

where P is the power in watt, V is the voltage across the conductor in volt, and I is the current through the conductor in amp. If we use Ohm's law to substitute for V in Equation (10) we obtain

$$P = (IR) \times I = I^2 R \tag{11}$$

Alternatively, if we rearrange Ohm's law as $I = V/R$ and substitute this expression for I in Equation (10) we get

$$P = V \times \left(\frac{V}{R}\right) \tag{12}$$

The three equivalent formulas for power dissipated in a single conductive element are

$$\boxed{P = VI = I^2 R = \frac{V^2}{R}} \tag{13}$$

Example 2 Energy consumed by a light bulb

How much energy is consumed when a 40W light bulb "burns" for 5 minutes?

Given: A 40W light bulb.

Find: Energy consumed in 5 minutes.

Solution: *Note: no voltage or current are given. Is there enough information to solve this problem?* Answer: yes.

The goal is to the find the amount of energy consumed. In engineering terminology, energy is an quantity independent of time, and power is the rate at which that energy is transferred (or dissipated). In other words, power is an amount of energy transfered in a given time interval.

We start with the defintion of power as

$$\text{Power} = \text{rate of energy transferred} = \frac{\text{energy transferred}}{\text{time}}$$

Let E be the amount of energy consumed by the bulb in time Δt.

$$P = \frac{E}{\Delta t} \quad \Longrightarrow \quad E = P\Delta t.$$

We know P and Δt. Therefore, the rest of the analysis amounts to substituting known values into the preceeding formula.

$$(40\,\mathrm{W})\left(5\,\mathrm{min} \times \frac{60\,\mathrm{s}}{1\,\mathrm{min}}\right) = \left(40\,\frac{\mathrm{J}}{\mathrm{s}}\right)(300\,\mathrm{s}) = 12000\,\mathrm{J} = 12\,\mathrm{kJ}.$$

Therefore,

$$\boxed{E = 12000\,\mathrm{J} = 12\,\mathrm{kJ}}.$$

Discussion: The analysis is a straight forward use of the definitions of power and energy. No knowledge of electrical circuits is used.

□

Example 3 Characteristics of a light bulb

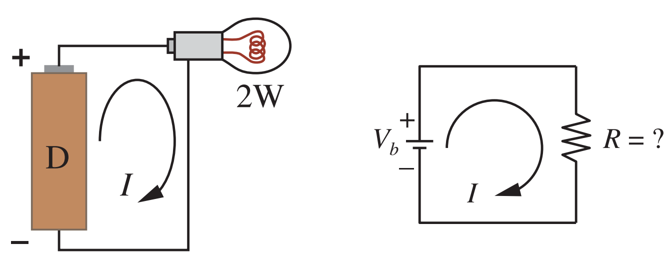
A D cell battery supplies 2W of power to a small lightbulb.

1. Find the current from the battery
2. Find the resistance of the bulb

Given: A D-cell connected in series with a light bulb.

Find: Current consumed and electrical resistance of the bulb if the battery supplies 2W.

Schematic:



Solution: What do we know?

> Battery is a D cell $\Longrightarrow V_b = 1.5\,\mathrm{V}$.
> Power dissipation is $2\,\mathrm{W}$.

We don't know the resistance of the light bulb.

Does Ohm's law apply? Yes!

$$V_b = IR$$

where I is the current flowing through the bulb, and R is the resistance of the bulb. Both I and R are unknown.

Additional information is available from the power dissipation of 2W. Apply the formula for power

$$P = V_b I$$

where both the power P and the battery voltage V_b are known. We can solve this equation for the unknown current, I

$$I = \frac{P}{V_b} = \frac{2\,\mathrm{W}}{1.5\,\mathrm{V}} = \frac{2}{3/2}\,\mathrm{A} = \frac{4}{3}\,\mathrm{A}.$$

Therefore

$$\boxed{I = \frac{4}{3}\,\mathrm{A}}$$

Now that I is known, use it in Ohm's law for the resistor

$$V_b = IR \Longrightarrow R = \frac{V}{I} = \frac{1.5\,\mathrm{V}}{\frac{4}{3}\,\mathrm{A}} = 1.125\,\Omega$$

Therefore

$$\boxed{R = 1.13\,\Omega}.$$

Discussion: There are two separate ways to solve this problem. We'll call these "Plan A" and "Plan B", respectively.

Plan A (used in the example) Start with

$$P = V_b I$$

where both P and V_b are known. Solve for I. With I known (freshly computed), apply Ohm's law

$$V_b = IR$$

Now that both V_b and I are known, we can solve for R.

$$R = \frac{V_b}{I}.$$

Plan B (Equally valid procedure:) Start with an alternative definition of power.

$$P = \frac{V_b^2}{R}$$

where both P and V_b are known. Solve for R

$$R = \frac{V_b^2}{R}$$

Now that R is known (freshly computed) we can apply Ohm's law

$$V_b = IR$$

and solve for I

$$\boxed{I = \frac{V_b}{R}}$$

It is not uncommon to have multiple algebraic paths in the analysis of an problem. In this case, both approaches require the same number of steps. That is not always the case. Although there is an advantage, or an aesthetic preference, for shorter and more direct solutions, the primary goal is mathematical correctness. An important, but secondary, goal is clarity – how well does the algebra show the logic of the analysis? In this case, both approaches are equivalent.

□

5 Equivalent Resistance

Consider the options for extending the circuit in Example 3 to circuits where the battery is connected to two light bulbs at the same time. There are two possible configurations as depicted in Figure 4. Are these configurations equivalent? For example, would you expect the brightness of the light bulbs in the two configurations to be the same?

Hands-on Learning: It would be a very good idea to gather some materials to experiment with these circuits. You will need a small flashlight bulb, an AA or AAA cell battery (or two), and a piece of scrap wire. The positive terminal of a typical incandescent light bulb is the nub that protrudes from the bottom end. The negative terminal is the metallic side of the bulb. The schematics in Figure 4 attempt to suggest these features and how to connect the circuits.

If you are using small flashlight bulbs and AA or AAA batteries, you can use your fingers to hold the bare ends of a small piece of wire to the battery terminals and bulbs. You could also use tape to make the connections more robust. More permanent solutions with glue or soldering are not necessary and would make it more difficult to experiment with alternate circuit configurations.

What happens when you connect the battery to the bulbs with the opposite polarity (i.e., switching the positive and negative terminals) from that shown in Figure 4? How does the brightness of the individual bulbs compare for the two arrangements in Figure 4?

Note that the use of incandescent light bulbs is a convenience for the purpose of demonstration. Any resistor, or more generally any *load*, such as a DC motor or resistive heating element, could be connected to the battery. Furthermore, the resistive elements need not be the same. We will explore the more general case of unequal resistances after discussing the simpler case of two equal resistive load elements.

What thoughts go through an engineer's mind when she looks at the two possible configurations of the light bulb circuits in Figure 4? An obvious question is, *Will either of these circuits work*? In other words, if I hook up the circuit according to either of the diagrams, will the two bulbs light up? The answer is, *it depends*. Try it! The two circuits in Figure 4 are correct in the sense that when the wires, batteries and bulbs are connected as shown, current will flow through the bulbs.

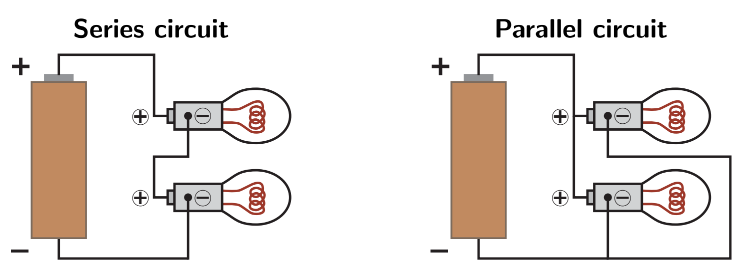


Figure 4: Two light bulbs arranged in a series connection (left) and a parallel connection (right).

Despite being correct circuits, there are practical choices of bulbs and batteries that will result in no light being emitted from the bulbs. How can that be? Being able to ask and then answer that question is an example of *thinking like an engineer*.

The light coming from the bulbs depends on the "size" of the battery and the "size" of the bulbs. We put size in quotes because that term is too vague for engineering purposes. We need to use the more precise language from the preceding sections of these notes. Instead of size, we need to think of voltage, current, resistance and power.

The size of the battery is specified by its *rated* voltage, current and power it can supply. By *rated* we mean the values that we expect from the label or the standard specifications for a given type of battery. The size of a light bulb is specified by its rated voltage and the amount of power the bulb can dissipate without burning out.

The question of whether either or both of the light bulb circuits work is determined by how well the power supplied by the battery matches the power that the light bulb can safely handle. If the battery cannot supply enough power, then the light bulbs will be dim, or may not even appear to emit any light. If the battery can supply too much power, then the bulb (or bulbs) may burn out. The question of how well the battery and the bulbs match is also affected by whether the series or parallel circuit is used. Therefore, the engineering question of whether the circuit "works" depends on the configuration of the wires *and* the choice of components (batteries and bulbs) used in the circuit.

Beyond the simple question of whether the circuit even works, there are several more interesting questions:

- How much current will the two bulbs draw?
- How long will the battery last?
- What happens if I have two batteries? And how should I connect the batteries – in series or parallel?
- For the two-bulb circuits, how does the brightness of each bulb compare to the case of only one bulb?
- How bright are the bulbs in each of the two-bulb configurations? In other words, for a given battery configuration, is it better (by some definition of "better") for the bulbs to be connected in series or parallel?

These are all practical questions. I encourage you to directly experiment with batteries and wires and bulbs to tie your hands-on experience to these notes. Above all, be safe! Small flashlight bulbs (2W or 4W) and 1.5V alkaline batteries (AAA, AA, A, C or D cell) will be fine.

5.1 Two Resistors in Series

The left side of Figure 5 shows the circuit diagram for two light bulbs (two resistors) in series. This is called a *series* circuit because all the current flowing from the battery goes first through one bulb and then the next. Refer to the left side of Figure 4 for the cartoon version of the circuit. The light bulbs are represented by resistors R_1 and R_2. *Incandescent* bulbs *are* resistors, so the diagram in the left side of Figure 5 is an excellent model of two incandescent light bulbs in series. The simple circuit would not apply to fluorescent lights or LEDs (Light emitting diodes) because those light-generating devices cannot be modeled as simple resistors.

The right side of Figure 5 shows the two resistors replaced by a single, equivalent resistance, R_{eq}. The value of R_{eq} results in the same current, I, flowing from the battery as for the two resisters in series.

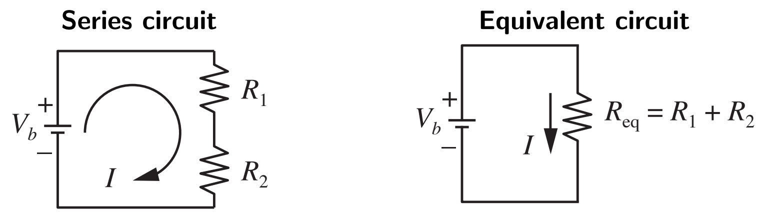


Figure 5: Two resistors in series (left) and the equivalent circuit (right).

For the circuit with two resistors in series, the current through each of the resistors is the same. In addition, the current leaving the battery is also the same as the current flowing through each resistor.

$$I = I_1 = I_2 \tag{14}$$

Apply Ohm's law to each resistor

$$V_1 = I_1 R_1 \qquad \text{and} \qquad V_2 = I_2 R_2 \tag{15}$$

Ohm's law also applies to the equivalent circuit on the right hand side of Figure 5

$$V_b = I R_{eq}. \tag{16}$$

The voltage across each battery is *not* the same as the voltage applied across each resistor. In fact, the *sum* of the voltages V_1 and V_2 must add up to V_b, the voltage supplied by the battery. This is a consequence of a more general principle called *Kirchoff's Voltage Law*, which we will discuss in detail later. For now, we can make the empirical observation that the two voltages add up to the total supplied voltage

$$V_b = V_1 + V_2. \tag{17}$$

We say that the voltage *drop* across the resistors must add up to the voltage drop across the two batteries. The word "drop" suggests that the voltage decreases in the direction of the current flow.

If we take Equation (17) as true (which it is!) then we can substitute the two forms of Ohm's law from Equation (15) and the version of Ohm's law for the equivalent circuit (Equation (16)) into Equation (17) to get

$$I R_{eq} = I_1 R_1 + I_2 R_2. \tag{18}$$

But, we know from Equation (14) that the current from the battery is the same as the current flowing through each of the resistors. Therefore, the I on the left hand side of Equation (18) is the same as both the I_1 and I_2 on the right hand side. Algebraically, because the I, I_1 and I_2 are equal (for *this* circuit!) those values can be cancelled from Equation (18) to give

$$\boxed{R_{eq} = R_1 + R_2}. \tag{19}$$

Equation (19) is true for any combination of two reistors in series.

5.2 Two Resistors in Parallel

The left side of Figure 6 shows the circuit diagram for two resistors in parallel. We say that this is a parallel circuit because the current from the battery splits to flow through each bulb separately, or in parallel. If the two bulbs have identical resistance, the current flow through each bulb will be equal. If the bulbs have unequal resistance, the current flowing will be unequal. Regardless of

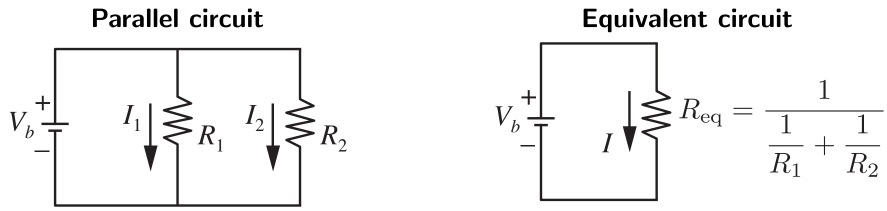


Figure 6: Two resistors in parallel (left) and the equivalent circuit (right).

whether the electrical currents have equal values, the electrons flowing through one bulb do not flow through the other bulb. The electrons from the battery split and flow on separate, parallel paths.

Refer to the right side of Figure 4 for the cartoon version of the circuit for two light bulbs wired in parallel. In the parallel circuit, the voltage across the battery is also the same as the voltage across each of the resistors

$$V_b = V_1 = V_2. \tag{20}$$

The relationship between the current flows is determined by *Kirchoff's Current Law*, which requires that the net flow of current into a node (or wire junction) be zero. In other words, the sum of the currents flowing *into* a node must be equal to the sum of currents flowing *out of* that same node.

Figure 7 shows the circuit for two resistors in parallel with the current flows labelled. The right side of Figure 7 shows the current flows into each of two nodes, labeled A and B where three currents I, I_1 and I_2 come together. For this circuit, it turns out that these nodes both give the same information, i.e., the same relationship between I, I_1 and I_2. In more complex circuits the relationships between current flows will involve different combinations of currents.

Applying Kirchoff's currently law to node A gives

$$I = I_1 + I_2 \tag{21}$$

and applying Kirchoff's current law to node B gives

$$I_1 + I_2 = I. \tag{22}$$

Algebraically, Equation (21) and (22) are equivalent. Therefore, we only need to analyze one of the nodes, either A or B, in this simple circuit.

If we rearrange Ohm's law as $I = V/R$ and apply it to each one of the terms in Equation (21) we obtain

$$\frac{V_b}{R_{eq}} = \frac{V_1}{R_1} + \frac{V_2}{R_2}. \tag{23}$$

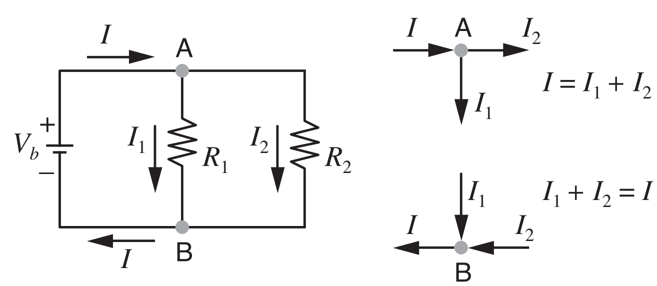


Figure 7: Current flows into nodes of the circuit for two resistors in parallel.

For the parallel circuit, all of the voltages are the same (See Equation (20).) Therefore, we can cancel the equal voltage terms in the numerators of Equation (23) to get

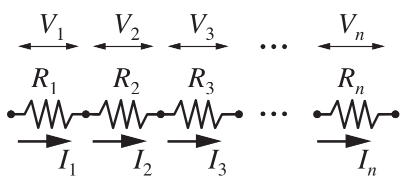
$$\boxed{\frac{1}{R_{\text{eq}}} = \frac{1}{R_1} + \frac{1}{R_2}.} \tag{24}$$

Equation (24) is true for any combination of two reistors in parallel.

5.3 General Rules for Resistors in Series and Parallel

The results from the preceding sections can be summarized and extended as follows.

Resistors in Series

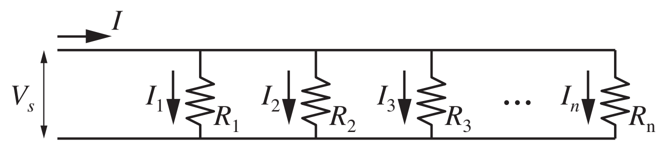


$$V_{\text{tot}} = V_1 + V_2 + V_3 + \cdots + V_n \quad \text{or} \quad V_{\text{tot}} = \sum_{i=1}^{n} V_i$$

$$I = I_1 = I_2 = I_3 = \cdots = I_n$$

$$R_{\text{eq}} = R_1 + R_2 + R_3 + \cdots + R_n \quad \text{or} \quad R_{\text{eq}} = \sum_{i=1}^{n} R_i$$

Resistors in Parallel



$$V_s = V_1 = V_2 = V_3 = \cdots = V_n$$

$$I = I_1 + I_2 + I_3 + \cdots + I_n$$

$$I_1 = \frac{V_s}{R_1}, \quad I_2 = \frac{V_s}{R_2}, \quad I_3 = \frac{V_s}{R_3}, \quad \cdots \quad I_n = \frac{V_s}{R_n}.$$

$$\frac{1}{R_{\text{eq}}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \cdots + \frac{1}{R_n} \quad \text{or} \quad \frac{1}{R_{\text{eq}}} = \sum_{i=1}^{n} \frac{1}{R_i}$$

The relationship between the currents is obtained by applying Kirchoff's current law to each node. There isn't a simple or compact formula that is the same for each node in the parallel resistor circuit.

6 Summary

1. Ohm's Law relates the flow of current to the voltage drop across a resistor.

   $$V = IR$$

   where V is the voltage (in volts, V), I is the current (in amp, A) and R is the resistance (in ohm, Ω).

   Notes:

   - V is used for both a voltage value and the units of volts.
   - A "resistor" can be any element that conducts electricity, e.g.,, a length of wire, a resistor in a circuit, or the filament of an incandescent light bulb.

2. At a basic physical level, current is defined as a flow of electrons

   $$1\,\text{A} = \frac{1\,\text{C}}{1\,\text{s}} = \frac{1\,\text{coulomb}}{1\,\text{second}}$$

   where coulomb is the unit of charge.

   At a basic physical level, voltage is the amount of energy associated with the separation of opposite charges

   $$1\,\text{V} = \frac{1\,\text{J}}{1\,\text{C}} = \frac{1\,\text{joule}}{1\,\text{coulomb}}$$

   In terms of resistivity, an intrinsic property of conductors, the macroscopic resistance of a wire (or any long and relatively skinny conductor) is

   $$R = \frac{\rho L}{A}$$

where ρ is the *resistivity*, L is the length of the wire, and A is the cross-sectional area of the wire.

3. Work and energy are quantities not dependent on time. Power is a rate.

   Work and energy can be defined in terms of raising a mass of material against the acceleration of gravity. The amount of mechanical work done in raising a mass, m, by a distance, d, is

$$\text{Work} = \text{force} \times \text{distance} = mgd.$$

   where g is the acceleration of gravity. The work done is equivalent to the change in potential energy of the mass

$$\text{Change in potential energy} = mgd.$$

   Power is the rate at which work is done.

$$\text{Power} = \frac{\text{work}}{\text{time}} = \frac{\text{energy}}{\text{time}}.$$

   Raising the mass m through a distance d always takes the same amount of work. However, raising it quickly takes more power than raising it slowly.

4. Power dissipated in a resistor can be computed by three equivalent formulas

$$P = VI, \qquad P = I^2R, \qquad P = \frac{V^2}{R}.$$

   We usually write these separate formulas in a single line as

$$P = VI = I^2R = \frac{V^2}{R}.$$

5. The equivalent resistance of two resistors in series is

$$\text{Two resistors in series:} \qquad R_{\text{eq}} = R_1 + R_2.$$

   The equivalent resistance of two resistors in parallel is

$$\text{Two resistors in parallel:} \qquad \frac{1}{R_{\text{eq}}} = \frac{1}{R_1} + \frac{1}{R_2}.$$

   Refer to Section 5.3 for cases involving more than two resistors.