A Breathing LED Indicator: Equations for PWM Output

ME 120 Notes

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Motivation

1. A reverse engineering exercise: emulate the breathing style of LED pulsing on a Macintosh laptop
2. Controlling LED brightness requires Pulse-width modulation (PWM), which can also used to control the speed of DC motors.
3. This is also an opportunity to practice algebra and Excel plotting

US Patent # 6,658,577 B2

US006658577B2

(12) **United States Patent**
Huppi et al.

(10) **Patent No.: US 6,658,577 B2**
(45) **Date of Patent: Dec. 2, 2003**

(54) **BREATHING STATUS LED INDICATOR**

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(*) Notice: Subject to any disclaimer, the term of this patent is extended or adjusted under 35 U.S.C. 154(b) by 0 days.

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(65) **Prior Publication Data**

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Related U.S. Application Data

(63) Continuation of application No. 09/332,242, filed on Jun. 14, 1999.

(51) **Int. Cl.**[7] **G06F 1/26**; G06F 1/28

(52) **U.S. Cl.** **713/323**; 713/320

(58) **Field of Search** 713/300–340

(56) **References Cited**

U.S. PATENT DOCUMENTS

5,608,225 A * 3/1997 Kamimura et al. 250/458.1
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Primary Examiner—Rupal Dharia

(57) **ABSTRACT**

A new and improved status LED indicator provides a pleasing visual appeal. An embodiment of the present invention includes a sleep-mode indicator for laptop computers. The LED indicator is energized by pulse-width modulated electrical pulses. The effect of these pulses on the indicator varies in intensity and mimics a rhythm typical of breathing. It is another aspect of the invention to provide an electrical apparatus that generates a sleep-mode indicator blinking pattern based on a sinusoidal function using PWM (pulse width modulation) designs.

9 Claims, 3 Drawing Sheets

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A new and improved status LED indicator provides a pleasing visual appeal. An embodiment of the present invention includes a sleep-mode indicator for laptop computers. The LED indicator is energized by pulse-width modulated electrical pulses. The effect of these pulses on the indicator varies in intensity and mimics a rhythm typical of breathing. It is another aspect of the invention to provide an electrical apparatus that generates a sleep-mode indicator blinking pattern based on a sinusoidal function using PWM (pulse width modulation) designs.

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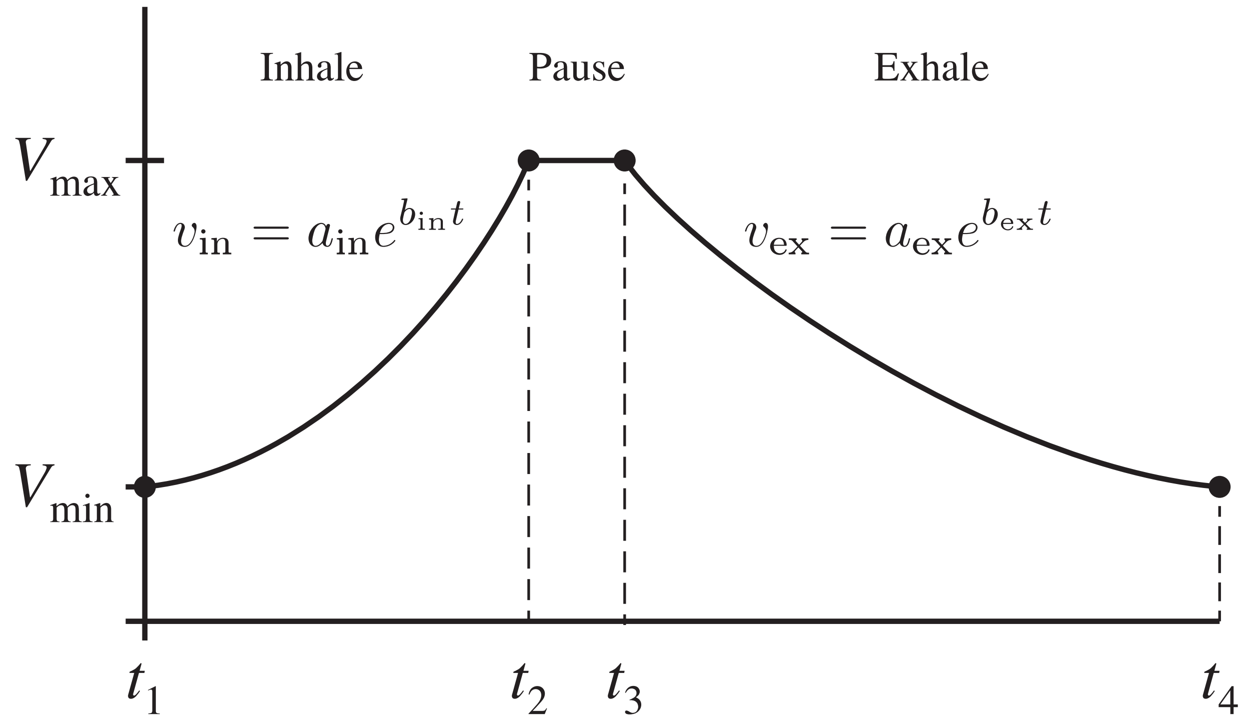
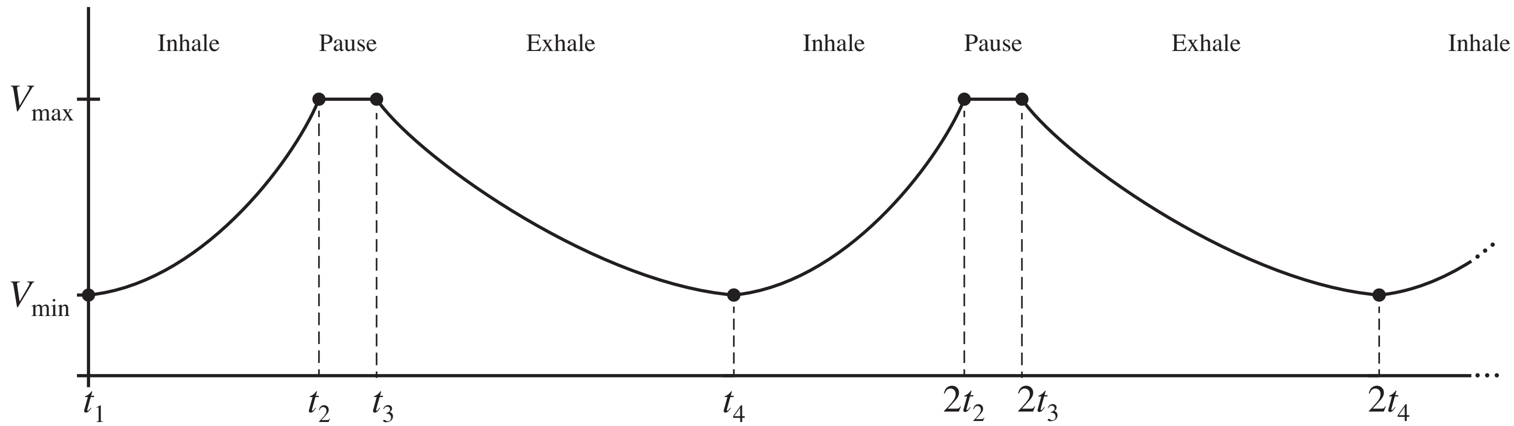
The breathing pattern has three phases: inhale, pause, and exhale



Note: This *not* the pattern claimed on US patent # 6658577.

The breathing pattern repeats indefinitely



The repeated pattern is the body of the `loop` function

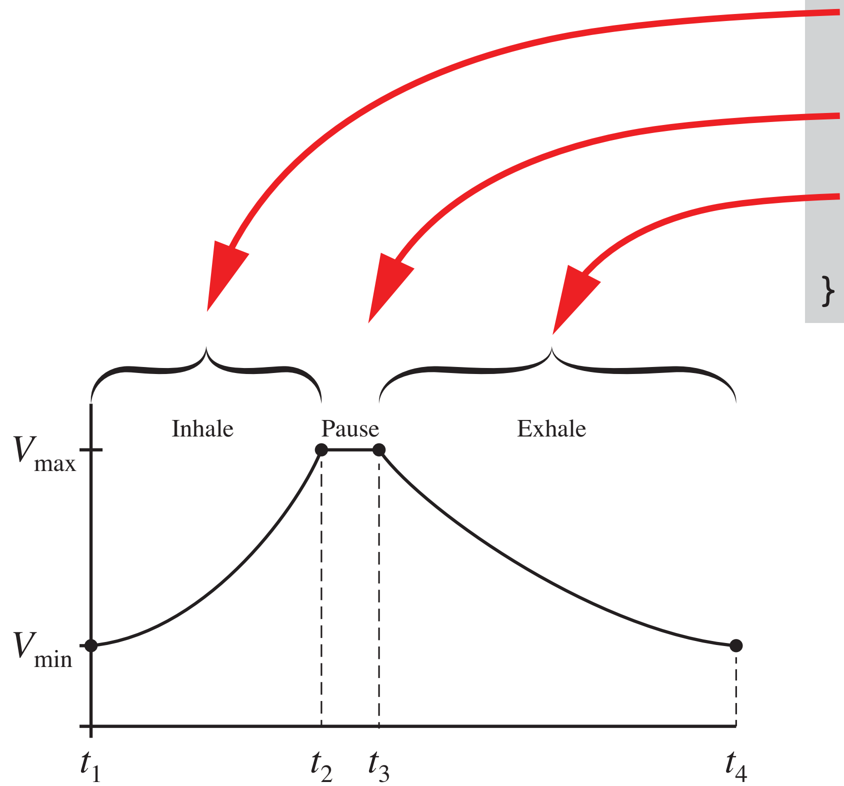
```
void loop() {

  // -- Inhale code

  // -- Pause code

  // -- Exhale code

}
```

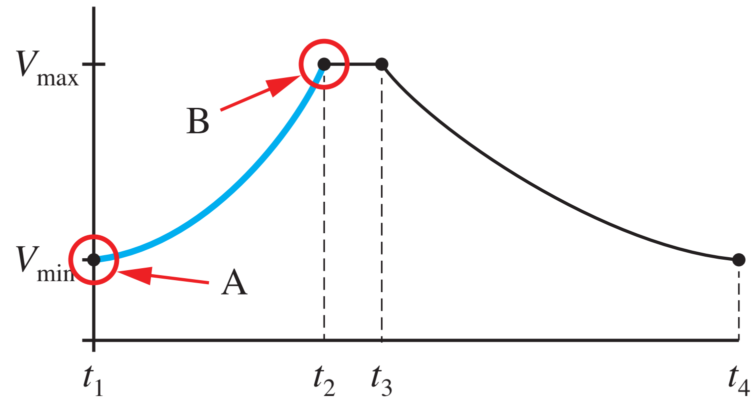


Inhale and exhale Functions have the same form

Inhale and exhale functions are of the form

$$v = ae^{bt} \tag{1}$$

Require that this function pass through two points (t_A, v_A) and (t_B, v_B).



Substituting these data pairs into Equation (1) gives

$$v_A = ae^{bt_A} \tag{2}$$

$$v_B = ae^{bt_B} \tag{3}$$

Linearize the Equations to Simply the Algebra

Recall that if

$$z = xy$$

then

$$\ln(z) = \ln(x) + \ln(y).$$

In words: the logarithm of a product is the sum of the logarithms of the terms being multiplied.

Also recall that if

$$r = e^{st}$$

then

$$\ln(r) = st.$$

Linearize the Equations to Simply the Algebra

Take the logarithm of Equation (1) and apply the rules for manipulating logarithms of products:

$$\ln(v) = \ln\left[ae^{bt}\right] \longrightarrow \ln(v) = \ln(a) + \ln\left[e^{bt}\right] \longrightarrow \ln(v) = \ln(a) + bt$$

Apply the transformation to Equations (2) and (3) to get

$$\ln(v_A) = \ln(a) + bt_A \tag{4}$$

$$\ln(v_B) = \ln(a) + bt_B \tag{5}$$

These are two *linear* equations for the two unknowns, $\ln(a)$ and b. The linear equations can be solved more easily.

Solve for a and b (1)

Subtract Equation (5) from Equation (4) to get

$$\ln(v_A) - \ln(v_B) = b(t_A - t_B) \tag{6}$$

Since t_A, v_A, t_B and v_B are known, we can solve for b

$$b = \frac{\ln(v_A) - \ln(v_B)}{t_A - t_B}. \tag{7}$$

Solve for a and b (2)

Now that the formula for b is known, we can substitute Equation (7) into either Equation (4) or Equation (5) to solve for a.

$$\ln(v_A) = \ln(a) + \left[\frac{\ln(v_A) - \ln(v_B)}{t_A - t_B}\right] t_A$$

$$\ln(a) = \ln(v_A) - \left[\frac{\ln(v_A) - \ln(v_B)}{t_A - t_B}\right] t_A$$

$$= \ln(v_A)\left[\frac{t_A - t_B}{t_A - t_B}\right] - \left[\frac{\ln(v_A) - \ln(v_B)}{t_A - t_B}\right] t_A$$

$$= \frac{\ln(v_A)\left[t_A - t_B\right] - \left[\ln(v_A) - \ln(v_B)\right] t_A}{t_A - t_B}$$

$$= \frac{t_A \ln(v_B) - t_B \ln(v_A)}{t_A - t_B}$$

Solve for a and b (3)

Therefore,

$$\ln(a) = \frac{t_A \ln(v_B) - t_B \ln(v_A)}{t_A - t_B}$$

Applying the exponential function to both sides of the preceding equation gives the formula for computing a

$$a = \exp\left[\frac{t_A \ln(v_B) - t_B \ln(v_A)}{t_A - t_B}\right] \tag{8}$$

We have created the formulas that model either inhale or exhale

Given (t_A, v_A) and (t_B, v_B), we can compute

$$a = \exp\left[\frac{t_A \ln(v_B) - t_B \ln(v_A)}{t_A - t_B}\right]$$

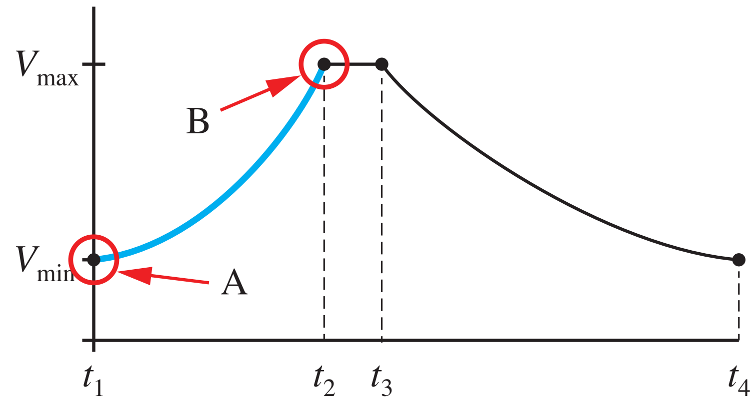
and

$$b = \frac{\ln(v_A) - \ln(v_B)}{t_A - t_B}.$$

which allows us to evaluate

$$v = ae^{bt}$$

for any t in the interval $t_A \leq t \leq t_B$.



Applying the Equations to find a and b (1)

1. Choose appropriate values of $V_{\min}$, $V_{\max}$, t_2, t_3 and t_4. These are somewhat arbitrary *design* choices that you make to achieve a desired look to your inhale and exhale functions.
2. Use Equations (8) and (7) to compute a_{in} and b_{in}.
3. Use Equations (8) and (7) (again) to compute a_{ex} and b_{ex}.

Applying the Equations to find a and b (2)

Once you have obtained values for a_{in}, b_{in}, a_{ex} and b_{ex}, it is a good idea to add this step

4. Plot the $v_{\text{in}}(t)$ and $v_{\text{ex}}(t)$ functions (say, with Excel or MATLAB) to make sure you do not have an error in your algebra.

Recapitulation

So far:

1. Both the inhale and exhale phases can be modeled with $v = ae^{bt}$
2. We choose the end points to give a desired shape.
3. When the endpoints are known, we can solve for a and b.
4. With known values of a and b for each phase, we can write code to control the brightness of the LED.

Next: translate the $v = ae^{bt}$ function into Arduino code.

But first: Let's make a plot of our $v = ae^{bt}$ functions to make sure we understand the math.