ME 120: DC Circuit Practice

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Simplifying Circuits with Multiple Resistors

The example problems provided here do not have numerical values specified for the battery voltage (V_b) or the resistors (R_1, R_2,...). The goal of these exercises is to focus on a procedure for simplifying arrangements of resistors, without being limited by any one set of numerical values. Therefore, the goal of the exercises is to obtain an algebraic expression that answers each of the questions.

For the circuits in Exercises 1 through 6,

1. What is the equivalent resistance of all resistors connected to the battery?
2. How much current is supplied by the battery?
3. How much power is dissipated by the circuit?
4. How much power is dissipated by R_1?

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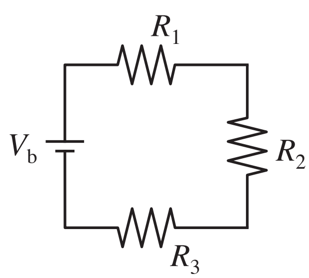
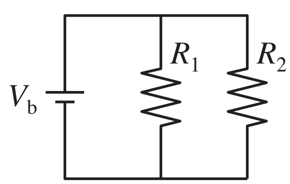
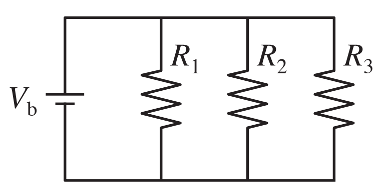
1. 

2. 

3. 

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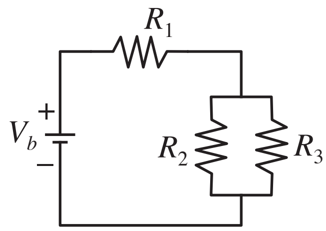

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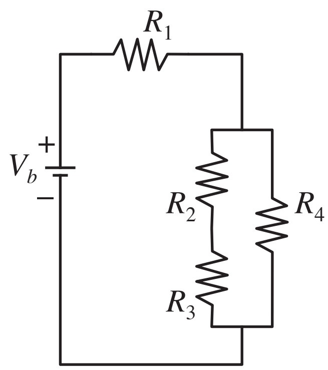

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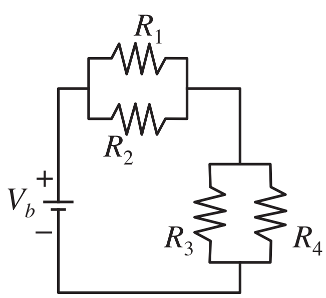
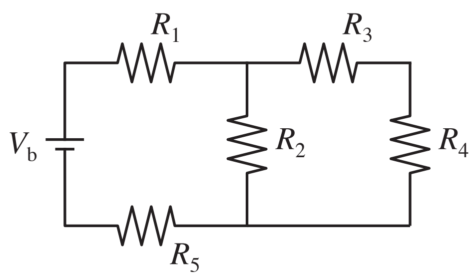


Simplifying Layouts of Resistor Networks

For each of the following circuits

- Transform the layout of the resistor network to an equivalent circuit where the resistors are at right angles, *and* where the network clearly shows which resistors are in series
- Develop a formula for the equivalent resistance of the network

7.



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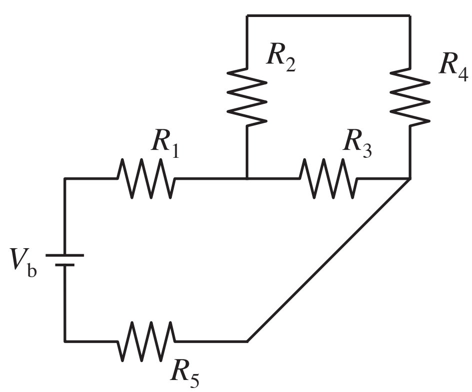

Solutions to Practice Problems

Simplifying Circuits with Multiple Resistors

The analysis of problems 1 through 6 follows these main steps:

a. Use rules for resistors in parallel and resistors in series to obtain an equivalent resistance, R_{eq}. With R_{eq} known, apply Ohm's law to compute I_{tot} from $V_b = I_{\text{tot}} R_{\text{eq}}$, where I_{tot} is the current supplied by the battery.

b. Compute the total power dissipated by the circuit with $P_{\text{tot}} = I_{\text{tot}}^2 R_{\text{eq}}$ or $P_{\text{tot}} = V_b I_{\text{tot}}$ or $P_{\text{tot}} = V_b^2 / R_{\text{eq}}$.

c. Computer P_1 in one of two ways

- Determine the current flowing through R_1 and compute $P_1 = I_1^2 R_1$, or
- Determine the voltage across R_1 and computer $P_1 = V_1^2 / R_1$.

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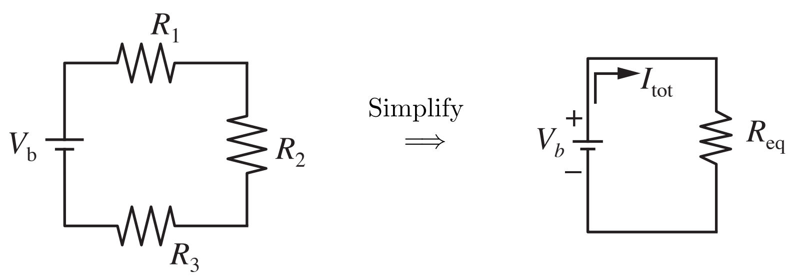
1. The circuit consists of three resistors in series. An equivalent resistance would cause the same current to flow from the battery.



a. The equivalent resistance of three resistors in series is

$$R_{\text{eq}} = R_1 + R_2 + R_3$$

The current supplied by the battery is computed from Ohm's law

$$V_b = I_{\text{tot}} R_{\text{eq}}$$

or

$$I_{\text{tot}} = \frac{V_b}{R_{\text{eq}}}$$

b. The *total* power dissipated by the circuit is

$$P_{\text{tot}} = I_{\text{tot}}^2 R_{\text{eq}}$$

c. The power dissipated by resistor R_1 is

$$P_1 = I_1^2 R_1$$

but, since $I_1 = I_{\text{tot}}$,

$$P_1 = I_{\text{tot}}^2 R_1$$

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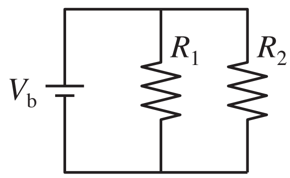
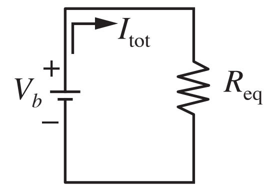
2. The circuit consists of two resistors in parallel. An equivalent resistance would cause the same current to flow from the battery.



Simplify
⟹



a. The equivalent resistance of two resistors in parallel is

$$R_{eq} = \frac{1}{\dfrac{1}{R_1} + \dfrac{1}{R_2}} = \frac{R_1 R_2}{R_1 + R_2}$$

Note: You should be able to use algebra to obtain the formula $R_{eq} = R_1 R_2/(R_1 + R_2)$ from the intermediate formula, $R_{eq} = 1/(1/R_1 + 1/R_2)$. The current supplied by the battery is computed from Ohm's law

$$V_b = I_{tot} R_{eq}$$

or

$$I_{tot} = \frac{V_b}{R_{eq}}$$

b. The *total* power dissipated by the circuit is

$$P_{tot} = I_{tot}^2 R_{eq}$$

c. The power dissipated by resistor R_1 can be computed with

$$P_1 = \frac{V_1^2}{R_1}$$

but, since $V_1 = V_b$,

$$P_1 = \frac{V_b^2}{R_1}$$

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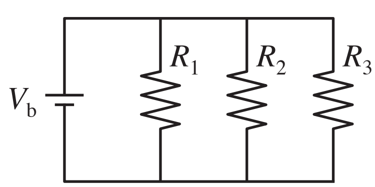
3. The circuit consists of three resistors in parallel. An equivalent resistance would cause the same current to flow from the battery.



Simplify
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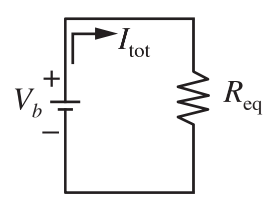

a. The equivalent resistance of three resistors in parallel is

$$R_{\text{eq}} = \frac{1}{\dfrac{1}{R_1} + \dfrac{1}{R_2} + \dfrac{1}{R_3}} = \frac{R_1 R_2 R_3}{R_2 R_3 + R_1 R_3 + R_1 R_2}$$

Note: You should be able to use algebra to obtain the formula $R_{\text{eq}} = R_1 R_2 R_2/(R_2 R_3 + R_1 R_3 + R_1 R_2)$ from the intermediate formula, $R_{\text{eq}} = 1/(1/R_1 + 1/R_2 + 1/R_3)$. The current supplied by the battery is computed from Ohm's law

$$V_b = I_{\text{tot}} R_{\text{eq}}$$

or

$$I_{\text{tot}} = \frac{V_b}{R_{\text{eq}}}$$

b. The *total* power dissipated by the circuit is

$$P_{\text{tot}} = I_{\text{tot}}^2 R_{\text{eq}}$$

c. The power dissipated by resistor R_1 can be computed with

$$P_1 = \frac{V_1^2}{R_1}$$

but, since $V_1 = V_b$,

$$P_1 = \frac{V_b^2}{R_1}$$

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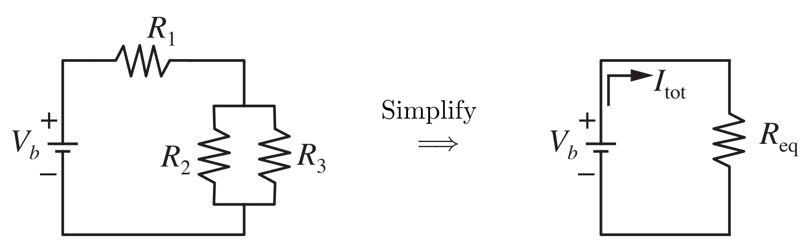
4. The circuit to be analyzed is shown below left. The first step of the analysis is to compute R_{eq} to obtain the equivalent circuit to the right.



a. Compute the total current supplied by the battery.

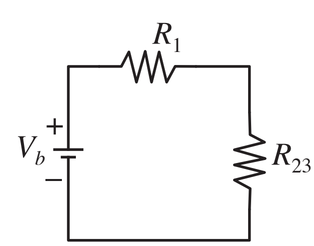
Step 1: Obtain a formula for the equivalent resistance, R_{eq} of the entire circuit, by replacing combinations of two (or more) resistors with an equivalent resistances. The process is repeated until there is only one equivalent resistance remaining.

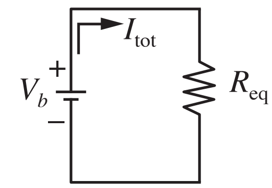
R_2 and R_3 are in parallel and can be combined into an equivalent resistance R_{23}

$$\frac{1}{R_{23}} = \frac{1}{R_2} + \frac{1}{R_3}$$

or, with algebraic rearrangement

$$R_{23} = \frac{R_2 R_3}{R_2 + R_3}$$





R_1 and R_{23} are in series and can be combined into an equivalent resistance R_{123}. Since R_{123} is the only remaining resistance, it is also R_{eq} for the entire circuit, i.e., $R_{123} = R_{eq}$.

$$R_{123} = R_{eq} = R_1 + R_{23}$$

Substituting the formula for R_{23}, gives

$$R_{eq} = R_1 + \frac{R_2 R_3}{R_2 + R_3} \qquad (\star)$$

The battery performs as if the three resistors in the given circuit are replaced by a single resistor with a restance value of R_{eq}.

Step 2: Compute the total current leaving the battery.

Ohm's law for the equivalent circuit is

$$V_b = I_{tot} R_{eq}$$

Therefore, since V_b and R_{eq} are known

$$I_{tot} = \frac{V_b}{R_{eq}} \qquad (\star\star)$$

This concludes part (a) of the assignment. Note that it is possible to substitute the detailed formula for R_{eq} in the preceding expression.

$$I_{tot} = \frac{V_b}{R_1 + \dfrac{R_2 R_3}{R_2 + R_3}}$$

However, this more complicated formula is cumbersome and adds little insight into the solution. If the values of R_1, R_2 and R_3 were known, then it would be better to compute R_{eq} as a numerical value from Equation ($\star$), above, and then use that numerical value in Equation ($\star\star$).

b. The power dissipated by the entire circuit can be computed from any of these formulas.

$$P_{tot} = I_{tot}^2 R_{eq} \quad \text{or} \quad P_{tot} = V_b I_{tot} \quad \text{or} \quad P_{tot} = \frac{V_b^2}{R_{eq}}.$$

From the givens, V_b and all the resistance values are known. From the analysis of part (a), R_{eq} is known. Thus, at this point in the analysis, all three of the formulas for P_{tot} can be computed directly.

c. The power dissipated by R_1 can be computed from any one of these formulas

$$P_1 = I_1^2 R_1 \quad \text{or} \quad P_1 = V_1 I_1 \quad \text{or} \quad P_1 = \frac{V_1^2}{R_1}.$$

Note that additional work would be required to find V_1. Since R_1 is in series with the battery,

$$I_1 = I_{tot} \qquad \text{therefore,} \qquad P_1 = I_{tot}^2 R_1$$

Also note that the preceding formula for P_1 *only* applies to the circuit being analyzed in this problem. $P_1 = I_{tot}^2 R_1$ is not true for all circuits because in general $I_1 \neq I_{tot}$.

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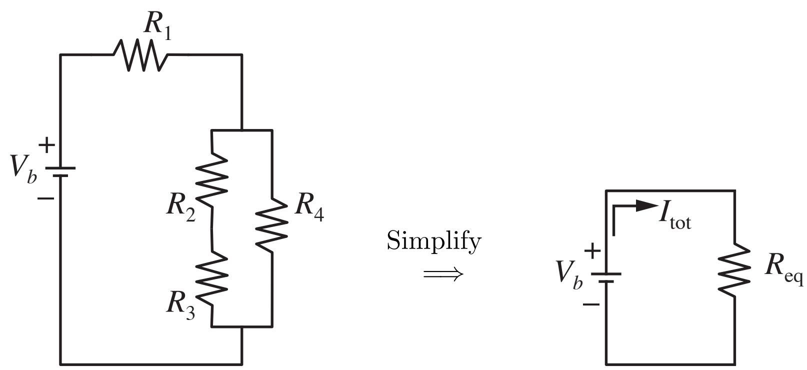
5. The circuit to be analyzed is shown below left. The first step of the analysis is to compute R_{eq} to obtain the equivalent circuit to the right.



The steps are the same as in the preceding exercise. However, since the arrangement of resistances is different, the formula for R_{eq} will be different.

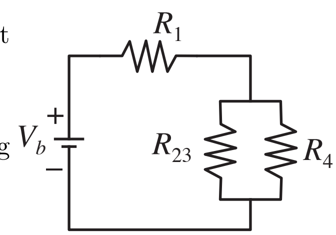
a. Compute the total current supplied by the battery.

Step 1: Obtain a formula for the equivalent resistance, R_{eq} of the entire circuit.

R_2 and R_3 are in series and can be combined into an equivalent resistance R_{23}

$$R_{23} = R_2 + R_3$$

Note that this is *not* the same R_{23} formula from the preceding exercise.

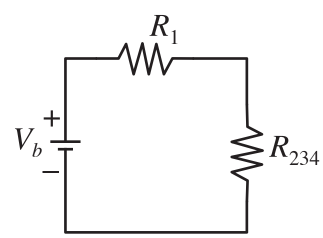


R_{23} and R_4 are in parallel and can be combined into an equivalent resistance R_{234}

$$\frac{1}{R_{234}} = \frac{1}{R_{23}} + \frac{1}{R_4}$$

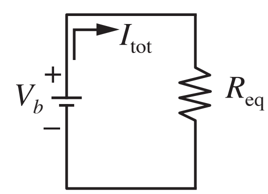
or, with algebraic rearrangement

$$R_{234} = \frac{R_{23}R_4}{R_{23} + R_4}$$



R_1 and R_{234} are in series and can be combined into an equivalent resistance R_{1234}. Since R_{1234} is the only remaining resistance, it is also R_{eq} for the entire circuit, i.e., $R_{1234} = R_{eq}$.

$$R_{1234} = R_{eq} = R_1 + R_{234}$$



It's possible to substitute the definitions of R_{234} and R_{23} into the expression for R_{eq}

$$R_{eq} = R_1 + R_{234} = R_1 + \frac{R_{23}R_4}{R_{23} + R_4} = R_1 + \frac{(R_2 + R_3)R_4}{R_2 + R_3 + R_4}$$

However, this more complicated expression is a bit unwieldy and it does not add much to our understanding of R_{eq}. In a problem in which numerical values of R_1, R_2, R_3 and R_4 are specified, it would be less cumbersome to substitute the numerical values into the formulas for R_{23} and R_{234} at this stage, and then use the numerical value of R_{eq} in subsequent computations.

Step 2: Compute the total current leaving the battery.

Ohm's law for the equivalent circuit is

$$V_b = I_{\text{tot}} R_{\text{eq}}$$

Therefore, since V_b and R_{eq} are known

$$I_{\text{tot}} = \frac{V_b}{R_{\text{eq}}}.$$

This concludes part (a) of this problem. Note that part (b) and part (c) of this problem are the same as part (b) and part (c) of the preceding problem.

b. The power dissipated by the entire circuit can be computed from any of these formulas.

$$P_{\text{tot}} = I_{\text{tot}}^2 R_{\text{eq}} \quad \text{or} \quad P_{\text{tot}} = V_b I_{\text{tot}} \quad \text{or} \quad P_{\text{tot}} = \frac{V_b^2}{R_{\text{eq}}}.$$

From the givens, V_b and all the resistance values are known. From the analysis of part (a), R_{eq} is known. Thus, at this point in the analysis, all three of the formulas can be computed directly.

c. The power dissipated by R_1 can be computed from any one of these formulas

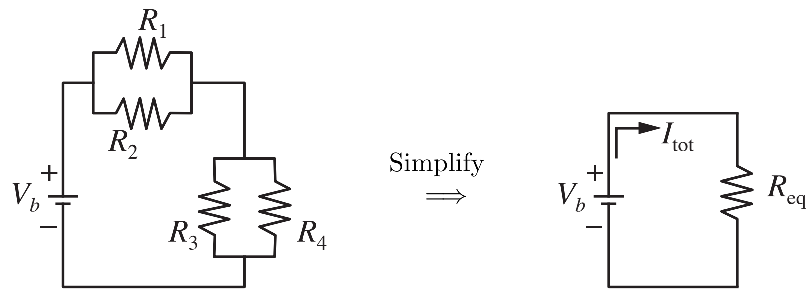
$$P_1 = I_1^2 R_1 \quad \text{or} \quad P_1 = V_1 I_1 \quad \text{or} \quad P_1 = \frac{V_1^2}{R_1}.$$

Note that additional work would be required to find V_1. Since R_1 is in series with the battery,

$$I_1 = I_{\text{tot}} \qquad \text{therefore,} \qquad P_1 = I_{\text{tot}}^2 R_1$$

$\Diamond$

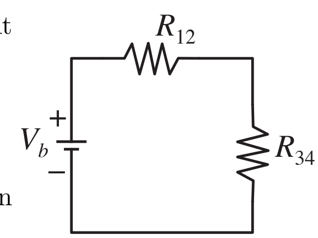
6. The circuit to be analyzed is shown below left. The first step of the analysis is to compute R_{eq} to obtain the equivalent circuit to the right.



The steps are the same as in the preceding exercise. However, since the arrangement of resistances is different, the formula for R_{eq} will be different.

a. Compute the total current supplied by the battery.
Step 1: Obtain a formula for the equivalent resistance, R_{eq} of the entire circuit.

R_1 and R_2 are in parallel and can be combined into an equivalent resistance R_{12}

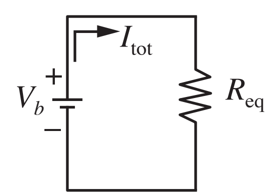


$$\frac{1}{R_{12}} = \frac{1}{R_1} + \frac{1}{R_2} \quad \text{or} \quad R_{12} = \frac{R_1 R_2}{R_1 + R_2}$$

Similarly, R_3 and R_4 are in parallel and can be combined into an equivalent resistance R_{34}

$$\frac{1}{R_{34}} = \frac{1}{R_3} + \frac{1}{R_4} \quad \text{or} \quad R_{34} = \frac{R_3 R_4}{R_3 + R_4}$$

R_{12} and R_{34} are in series and can be combined into an equivalent resistance R_{1234}. Since R_{1234} is the only remaining resistance, it is also R_{eq} for the entire circuit, i.e., $R_{1234} = R_{\text{eq}}$.



$$R_{1234} = R_{\text{eq}} = R_{12} + R_{34}$$

We can substitute the definitions of R_{12} and R_{34} into the expression for R_{eq} to get

$$R_{\text{eq}} = \frac{R_1 R_2}{R_1 + R_2} + \frac{R_3 R_4}{R_3 + R_4}$$

Once again, this slightly unwieldy formula is not more instructive than the equivalent form, $R_{\text{eq}} = R_{12} + R_{34}$. At this point in an analysis with known values for R_1, R_2, R_3 and R_4, it would be better to substitute those values into the formula for R_{eq}, and then treat R_{eq} as a known numerical value.
Step 2: Compute the total current leaving the battery.
Ohm's law for the equivalent circuit is

$$V_b = I_{\text{tot}} R_{\text{eq}}$$

Therefore, since V_b and $R_{\rm eq}$ are known

$$I_{\rm tot} = \frac{V_b}{R_{\rm eq}}$$

This concludes part (a) of this problem. Note that part (b) of this problem are the same as part (b) of the preceding problem. Part (c) requires some additional analysis

b. The power dissipated by the entire circuit can be computed from any of these formulas.

$$P_{\rm tot} = I_{\rm tot}^2 R_{\rm eq} \quad \text{or} \quad P_{\rm tot} = V_b I_{\rm tot} \quad \text{or} \quad P_{\rm tot} = \frac{V_b^2}{R_{\rm eq}}.$$

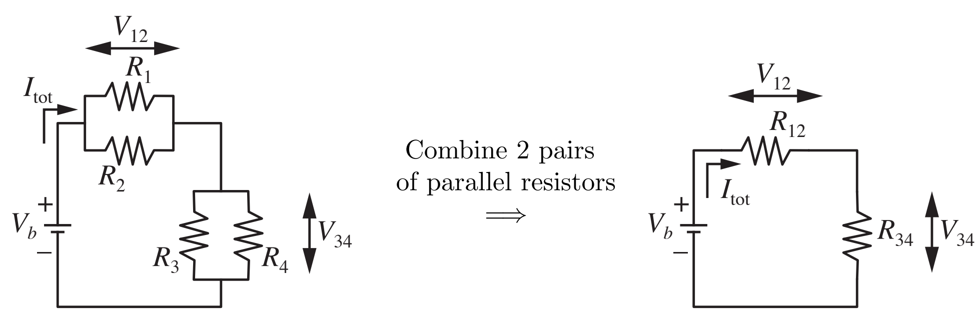
From the givens, V_b and all the resistance values are known. From the analysis of part (a), $R_{\rm eq}$ is known. Thus, at this point in the analysis, all three of the formulas can be computed directly.

c. The power dissipated by R_1 can be computed from one of these formulas.

$$P_1 = I_1^2 R_1 \quad \text{or} \quad P_1 = V_1 I_1 \quad \text{or} \quad P_1 = \frac{V_1^2}{R_1}.$$

At this point in the analysis, only the given value of R_1 is known. Calculation of V_1 or I_1 will require additional effort. Note that if V_1 was known, it would be easy to compute I_1 from Ohm's law, viz., $I_1 = V_1/R_1$. However, if V_1 was known, it would quicker to directly compute $P_1 = V_1^2/R_1$. We will use Kirchoff's voltage law to find V_1.

Below left is the original circuit diagram with identification of the voltage drops V_{12} and V_{34}, and the total current leaving the battery, $I_{\rm tot}$. Below right is the same circuit after one degree of simplification that was achieved by combining R_1 and R_2 into the equivalent resistance R_{12}, and by combining R_3 and R_4 into the equivalent resistance R_{34}.



Because R_1 and R_2 are in parallel, the voltage drop, V_{12} is the same as the voltage drop for R_1 and for R_2.

$$V_{12} = V_1 = V_2$$

Therefore, (for this circuit!), by finding V_{12} we will know V_1.

From the preceding analysis, $I_{\rm tot} = V_b/R_{\rm eq}$ is known. Applying Ohm's law to the equivalent resistance R_{12} gives

$$V_{12} = I_{\rm tot} R_{12}$$

Since $I_{\rm tot}$ and R_{12} are known, the preceding formula gives the value for V_{12}. Therefore

$$P_1 = \frac{V_1^2}{R_1} = \frac{V_{12}^2}{R_1} = \frac{(I_{\rm tot} R_{12})^2}{R_1}$$

The terms in the last expression on the right hand side are computable from the given constants of the problem (V_b, R_1, R_2, R_3 and R_4).

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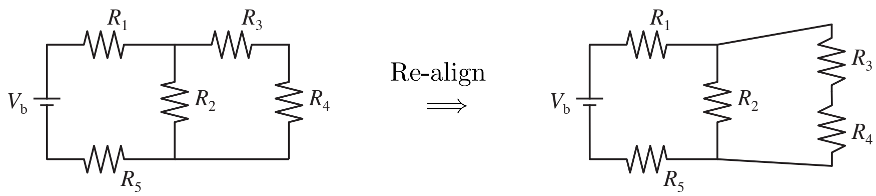
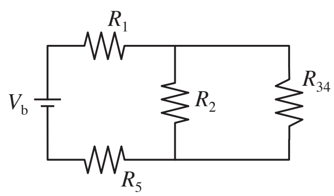
7. R_3 and R_4 are in series. R_2 is *not* in parallel with R_4. Stretching and rearranging the wires connecting R_3 and R_4 transforms the layout (but not the fundamental behavior of the circuit).

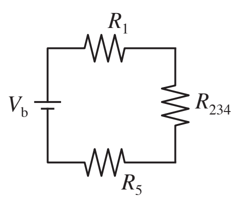


R_3 and R_4 are in series and can be replaced with R_{34}



$$R_{34} = R_3 + R_4$$

R_2 and R_{34} are in parallel, and can be combinted to give R_{234}



$$R_{234} = \frac{R_2 R_{34}}{R_2 + R_{34}}$$

The last step is to combine R_1, R_{234} and R_5.

$$R_{\text{eq}} = R_1 + R_{234} + R_5.$$

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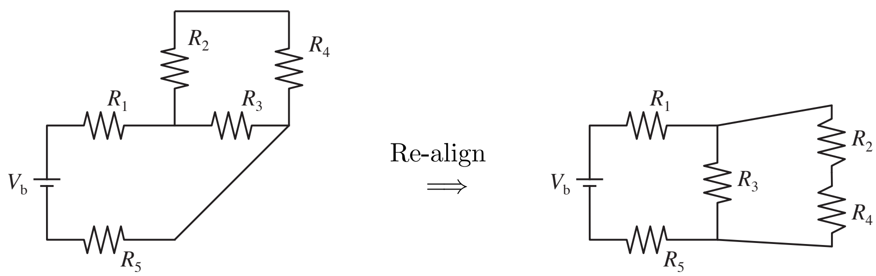
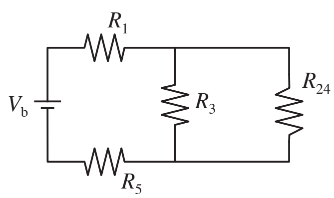
8. This circuit has the same topology as the preceding circuit, but with the roles of R_2 and R_3 switched.

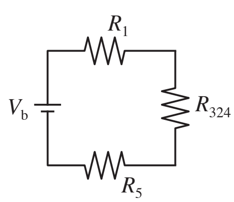


R_2 and R_4 are in series.



$$R_{24} = R_2 + R_4$$

R_3 and R_{24} are in parallel, and can be combinted to give R_{324} (or, if you prefer, R_{234}).



$$R_{324} = \frac{R_3 R_{24}}{R_3 + R_{24}}$$

The last step is to combine R_1, R_{324} and R_5.

$$R_{\text{eq}} = R_1 + R_{324} + R_5.$$

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