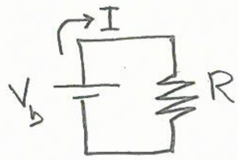
Build a decoration with multiple incandescent lights.
Should the light bulbs be wired in series or parallel?
We'll answer this question by considering just two bulbs.

<u>Baseline</u>: one bulb connected to one AA (1.5V) battery



$V_b = 1.5V$ given

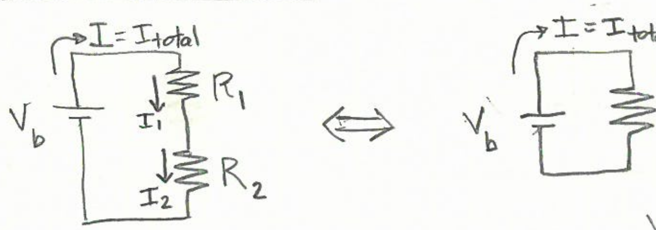
$R \sim 1.5\Omega$ measured

$V_b = IR \Rightarrow I = \frac{V_b}{R} = 1A$

$P = I^2 R = (1A)^2(1.5\Omega) = 1.5W$

$I = 1A$
$P = 1.5W$

<u>Two bulbs in series</u>: Assume that all bulbs have the same resistance, R



$R_{eq} = R_1 + R_2 \quad R_1 = R_2 = R \Rightarrow R_{eq} = 2R$

$V_b = IR_{eq} \Rightarrow I = \frac{V_b}{R_{eq}} = \frac{1.5V}{(2)(1.5\Omega)} = $ **$0.5A = I_{total}$**

$P = I^2 R_{eq} = (0.5A)^2(3.0\Omega) = $ **$0.75W = P_{total}$**

Power dissipation in individual resistors

$P_1 = I^2 R_1 = (0.5A)^2(1.5\Omega) = 0.375W$

$P_2 = I R_2 = (0.5A)^2(1.5\Omega) = 0.375W$

} Note current I is the same through both resistors

Check: Is the total power calculated with R_{eq} equal to the sum of power consumed by the individual resistors?

$P_{total} \overset{?}{=} P_1 + P_2$

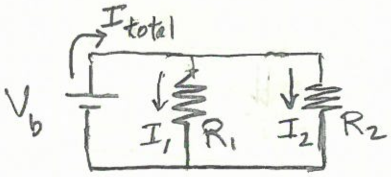
$0.75W \overset{?}{=} 0.375W + 0.375W$

$0.75W = 0.75W$ yes!

Summary for two resistors (bulbs) in series

$I_{total} = I = 0.5A$
$P_{total} = 0.75W$

Two bulbs in Parallel: Again, assume that both bulbs have the same resistance



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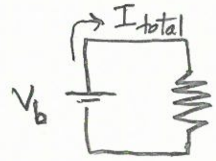

$$R_{eq} = \frac{R_1 R_2}{R_1 + R_2}$$ (see notes for derivation)

$$R_1 = R_2 = R \Rightarrow R_{eq} = \frac{R}{2}$$

Because the current is split between the two resistors, we need to distinguish I_{total}, the current leaving the battery from the separate currents, I_1 and I_2, that flow through the individual resistors. Soon we will talk about Kirchoff's current law which requires $I_{total} = I_1 + I_2$

$$R_{eq} = \frac{1.5\Omega}{2}$$

$$\boxed{R_{eq} = 0.75\Omega}$$

Apply Ohm's law: $V_b = I_{total} R_{eq} \Rightarrow I_{total} = \frac{V_b}{R_{eq}} = \frac{2V_b}{R}$

$$I_{total} = \frac{(2)(1.5V)}{(1.5\Omega)} = 2A \qquad \boxed{I_{total} = 2A}$$

The total power supplied by the battery is

$$P_{total} = I_{total}^2 R_{eq} = (2A)^2(0.75\Omega) = 3W$$

$$\boxed{P_{total} = 3W}$$

Power dissipation in individual resistors:

We don't yet know the current flowing through the individual resistors, but we can use the $P = \frac{V^2}{R}$ formula on each resistor because each resistor has a voltage drop of V_b, which is known.

$$P_1 = \frac{V_1^2}{R_1} = \frac{V_b^2}{R_1} = \frac{(1.5V)^2}{(1.5\Omega)} = 1.5W$$

$$P_2 = (\text{same as } P_1) = \frac{V_b^2}{R_2} = \frac{(1.5V)^2}{(1.5\Omega)} = 1.5W$$

Check power totals: $P_{total} \overset{?}{=} P_1 + P_2$

$$3W \overset{?}{=} 1.5W + 1.5W \quad \text{yes!}$$

Summary and Comparison of wiring configurations

	I_{total}	I_1	I_2	P_{total}	P_1	P_2
Baseline (one bulb)	1A	1A	N.A	1.5W	1.5W	N.A
Bulbs in Series	0.5A	0.5A	0.5A	0.75W	0.375W	0.375W
	(note: $I_{total} = I_1 = I_2 \neq I_1 + I_2$)					
Bulbs in parallel	2A	1A	1A	3W	1.5W	1.5W
	(note $I_{total} = I_1 + I_2$)					

Observations

- Putting two bulbs in series increases the equivalent resistance for the battery, which reduces the current it delivers. Each of the two bulbs dissipates less power than the baseline, which means that the bulbs will appear to be less bright
- Putting two bulbs in parallel reduces the equivalent resistance for the battery, which increases the current it delivers. Each of the two bulbs dissipates the same power as the base case single bulb. Therefore the total power is twice as large as the base case
- In all cases, the total power consumption is equal to the sum of power consumed by the individual bulbs
- The analysis ignores the real limit of the battery to supply power. Adding bulbs in parallel increases the theoretical power delivered by the battery. Real batteries have finite capacity for delivering power