Voltage Dividers and Potentiometers

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1 Introduction

Voltage dividers and potentiometers are passive circuit components that provide a simple way to convert a DC voltage level to another, lower, DC voltage level. Figure 1 shows the electrical circuit of a voltage divider on the left, and a potentiometer on the right.

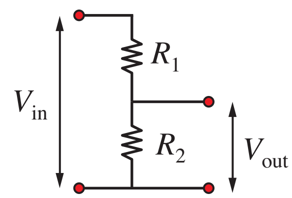


Figure 1: A voltage divider on the left, and potentiometer on the right.

A voltage divider consists of two resistors in series with a voltage tap between the two resistors. In the left side of Figure 1, the input voltage, V_{in}, is applied across R_1 and R_2. The output voltage, V_{out}, is the voltage drop across R_2. V_{out} is less than V_{in} because the total voltage across R_1 and R_2 must add up to V_{in}.

A potentiometer is a voltage divider that allows adjustment of V_{out}. Typical potentiometers have sliders or rotary knobs that move a contact called a *wiper* along the surface of a resistor. As depicted in the right side of Figure 1, the wiper divides a single, fixed resistor into R_1 and R_2. By sliding the wiper along the fixed resistor, the value of R_2 is changed, which allows the output voltage to be adjusted from 0 to V_{in}.

Voltage dividers and potentiometers are passive in the sense that they transform V_{in} to V_{out} without a separate source of power. Any power consumed during the transformation comes from the source of the input voltage. In contrast, an active component requires an external source of power to operate. Because voltage dividers and potentiometers are passive, these devices can only decrease the voltage, i.e., V_{out} is always less than V_{in}. Boosting the voltage from V_{in} to a higher level V_{out} requires an amplifier, which is an active component.

2 Analysis of a Voltage Divider

Applying Kirchoff's voltage law and Ohm's law, we can obtain a simple formula for V_{out} as a function of V_{in}, R_1 and R_2 for the two circuits in Figure 1. The resulting formula applies to either a voltage divider or a potentiometer, the two devices are electrically equivalent.

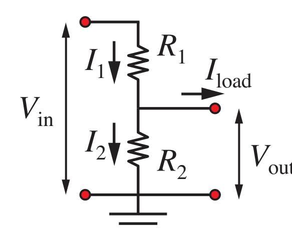


Figure 2: Current flows in a voltage divider.

Figure 2 shows the possible current flows in a voltage divider. For convenience, the bottom nodes of the circuit have been tied to ground. The current flows and other operating characteristics of the voltage divider do not change if the lower nodes are not grounded. The formulas derived in this section do not require the lower nodes to be grounded.

The current flowing out of the voltage divider is called the *load* current, I_{load}. The current leaving the voltage divider (as opposed to flowing straight to ground) is a load current in the sense that the electrical power is put some use, such as powering a DC motor, an LED, or some other device that consumes electrical energy. The analysis of the voltage divider or potentiometer is separated into two cases: one where I_{load} is negligible, and one where I_{load} needs to be included in the analysis.

Apply Kirchoff's current law to the node between the two resistors in the voltage divider depicted in Figure 2.

$$I_1 = I_{\text{load}} + I_2 \tag{1}$$

where I_1 and I_2 are the currents flowing through R_1 and R_2, respectively.

The load current is negligible in applications where the purpose of V_{out} is to provide a reference voltage that controls the operation of another (usually active) electronic device. Potentiometers can be used to provide user input to the operation of an Arduino microcontroller. For example, a potentiometer could adjust a voltage level that an Arduino uses to determine how fast to blink an LED.

2.1 Case of Infinite Load Resistance: $I_{\text{load}} = 0$

If the load resistance is infinite, I_{load} will be zero. There are practical applications where I_{load} is so small that its affect on the operation of the voltage divider is negligible. Figure 3 shows the current flow in a voltage divider with an infinite load resistance.

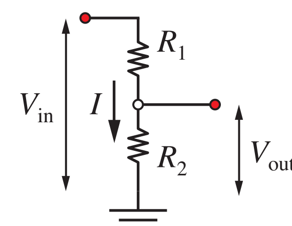


Figure 3: Current flows in a voltage divider when $I_{\text{load}} = 0$.

If we assume that the load current is negligible, then Equation (1) shows that $I_1 = I_2$. When $I_{\text{load}} = 0$ all the current supplied by the voltage source V_{in} flows to ground through the series combination of R_1 and R_2. Let that current be called I. In other words, $I_1 = I_2 = I$ as shown in Figure 3.

Apply Ohm's law to the series combination of R_1 and R_2 with the common current I

$$\begin{aligned} V_{\text{in}} &= IR_{\text{eff}} \\ &= I(R_1 + R_2) \end{aligned} \tag{2}$$

where $R_{\text{eff}} = R_1 + R_2$ is the effective resistance of R_1 and R_2 in series. Solve Equation (2) for I

$$I = \frac{V_{\text{in}}}{R_1 + R_2} \tag{3}$$

Apply Ohm's law to R_2

$$V_{\text{out}} = IR_2. \tag{4}$$

Combine Equation (3) and Equation (4) to eliminate I

$$V_{\text{out}} = V_{\text{in}} \frac{R_2}{R_1 + R_2}. \tag{5}$$

Equation (5) is the commonly used equation for the voltage output of a voltage divider. It applies to the case where $I_{\text{load}} = 0$. Analysis in Section 2.3 shows that this simple formula is adequate in most cases.

2.2 Case of Finite Load Resistance: $I_{\text{load}} \neq 0$

A voltage divider (or potentiometer) can still operate in situations where the load current is not negligible. However, when I_{load} is not small, Equation (5) is not an accurate prediction of V_{out}. Figure 4 shows the circuit diagram for a voltage divider with non-negligible I_{load}. R_3 is the load resistance.

The effective resistance of the circuit in Figure 4 is

$$R_{\text{eff}} = R_1 + \frac{R_2 R_3}{R_2 + R_3} \tag{6}$$

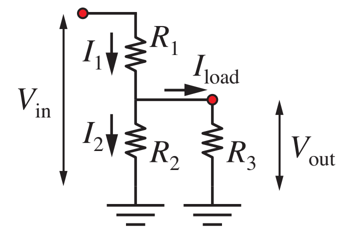


Figure 4: A voltage divider with non-negligible load current.

and the total current is

$$I = \frac{V_{\text{in}}}{R_{\text{eff}}}. \tag{7}$$

Kirchoff's voltage law requires

$$V_{\text{in}} = V_1 + V_{\text{out}}$$

where V_1 is the voltage drop across R_1 and V_{out} is the voltage drop across the parallel combination of R_2 and R_3. Solve the preceding equation for V_{out} to get

$$V_{\text{out}} = V_{\text{in}} - V_1. \tag{8}$$

Apply Ohm's law to R_1 to obtain an equation for V_1

$$V_1 = IR_1. \tag{9}$$

Substitute Equation (7) for I into the preceding equation to find a relationship between V_1 and V_{in}

$$V_1 = \frac{V_{\text{in}}}{R_{\text{eff}}} R_1 = V_{\text{in}} \frac{R_1}{R_{\text{eff}}}. \tag{10}$$

Substitute this expression for V_1 into Equation (8) to get

$$V_{\text{out}} = V_{\text{in}} - V_{\text{in}} \frac{R_1}{R_{\text{eff}}} = V_{\text{in}} \left[1 - \frac{R_1}{R_{\text{eff}}} \right] \tag{11}$$

Substitute the formula for R_{eff} from Equation (6) and use algebra to simplify the resulting equation to obtain

$$V_{\text{out}} = V_{\text{in}} \frac{R_2}{R_1 \left(\dfrac{R_2}{R_3} + 1 \right) + R_2}. \tag{12}$$

Refer to the Appendix for the intermediate algebraic steps. Equation (12) is the formula for the voltage divider when the load current is not negligible. Remember that R_3 is the resistance of the load, which is a characteristic of the system to which the voltage divider is attached, not a part of the voltage divider itself. When $R_3 \to \infty$, the ratio $R_2/R_3 \to 0$ and Equation (12) simplifies to Equation (5).

2.3 Comparing the Infinite and Finite Load Resistance Cases

The analysis in Section 2.1 and Section 2.2 provide two models for the voltage output of a voltage divider. The case of infinite load resistance in Section 2.1 gives a simpler formula obtained by ignoring the load resistance altogether. The case of finite load resistance in Section 2.2 is more general and more accurate, especially when R_3 is not that much larger than R_2. This begs the question: to what degree and in what circumstances does a finite R_3 matter? We will answer that question with two different models.

First, consider the case where the load resistance is not infinite and when the two resistors in the voltage divider are equal, i.e., $R_1 = R_2$. Begin by rearranging Equation (11) slightly,

$$\frac{V_{\text{out}}}{V_{\text{in}}} = \frac{R_2}{R_1\left(\dfrac{R_2}{R_3} + 1\right) + R_2} \tag{13}$$

Now, set R_1 equal to R_2 and simplify

$$\frac{V_{\text{out}}}{V_{\text{in}}} = \frac{1}{\left(\dfrac{R_2}{R_3} + 1\right) + 1} = \frac{1}{\dfrac{R_2}{R_3} + 2} \qquad \text{Special case: } R_1 = R_2 \tag{14}$$

Remember that Equation (14) *only* applies when $R_1 = R_2$. Table 1 shows an example of applying Equation (14) when $R_1 = R_2 = 10\,\text{k}\Omega$ for a sequence of decreasing R_3 values. When R_3 is large, e.g., for $R_3/R_2 = 1000$ or $R_3/R_2 = 100$ in Table 1, the value of $V_{\text{out}}/V_{\text{in}}$ is very close to value of 0.5 that is obtained by ignoring R_3 for the case of $R_1 = R_2$.

Now, consider a more general analysis of the role of R_3 without the restriction that $R_1 = R_2$. Figure 5 shows a potentiometer circuit with the output connected across a load resistor R_3. The values of R_1 and R_2 are adjusted by changing the position of the potentiometer wiper. For convenience, let R_t be the total resistance of the potentiometer

$$R_t = R_1 + R_2 \tag{15}$$

and let α be the fractional position of the potentiometer wiper

$$\alpha = \frac{R_2}{R_t}. \tag{16}$$

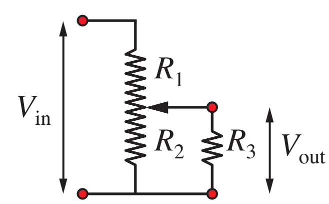


Figure 5: Potentiometer circuit with a non-zero load resistor, R_3.

Table 1: Effect of the load resistance on output voltage of a voltage divider with $R_1 = R_2 = 10\,\text{k}\Omega$. If the simpler formula, Equation (5), for the voltage divider is used, i.e., if we instead assume that $R_3/R_2 = \infty$, we expect to obtain $V_{\text{out}}/V_{\text{in}} = 0.5$.

R_3	R_3/R_2	$V_{\text{out}}/V_{\text{in}}$
$10\,\text{M}\Omega$	1000	0.4998
$1\,\text{M}\Omega$	100	0.498
$100\,\text{k}\Omega$	10	0.476
$10\,\text{k}\Omega$	1	0.333

Note that when $R_3 = \infty$, $V_{\text{out}}/V_{\text{in}} = \alpha$. Substituting Equation (16) into Equation (13) and simplifying gives

$$\frac{V_{\text{out}}}{V_{\text{in}}} = \frac{\alpha R_t}{(1-\alpha)R_t\left(\dfrac{\alpha R_t}{R_3}+1\right)+\alpha R_t}$$

$$= \frac{\alpha}{(1-\alpha)\left(\dfrac{\alpha R_t}{R_3}+1\right)+\alpha} \tag{17}$$

The preceding equation shows that the potentiometer output is determined by two parameters: the position of the wiper, α, and the ratio of the total potentiometer resistance to the load resistance, R_t/R_3. Usually we consider the case where $R_t/R_3 < 1$, i.e., where R_3 is large.

Figure 6 is a plot of Equation (17), with α on the horizontal axis and R_3/R_t as the parameter. The solid line corresponds to $R_3 = \infty$, i.e., the simple potentiometer model where the load resistance is so large that it is not important. For a given α, points A and B shows the range of $V_{\text{out}}/V_{\text{in}}$ when R_3 varies from ∞ to R_t. For a given, desired V_{out}/Vin, the points A and C show that α needs to change (the wiper needs to be moved) when R_3 is not large enough to be neglected.

The analysis in this section shows that the load resistance is not a big obstacle to using a potentiometer. The computations summarized in Table 1 demonstrate that with $R_3/R_2 > 100$, the effect of the load resistance on the output voltag is negligible. Figure 6 shows that even when $R_3/R_t = 1$, the effect of the output resistor can be compensated by making a modest adjustment to the potentiometer. We conclude that the simpler potentiometer formula, Equation (5), can be used for most engineering design calculations.

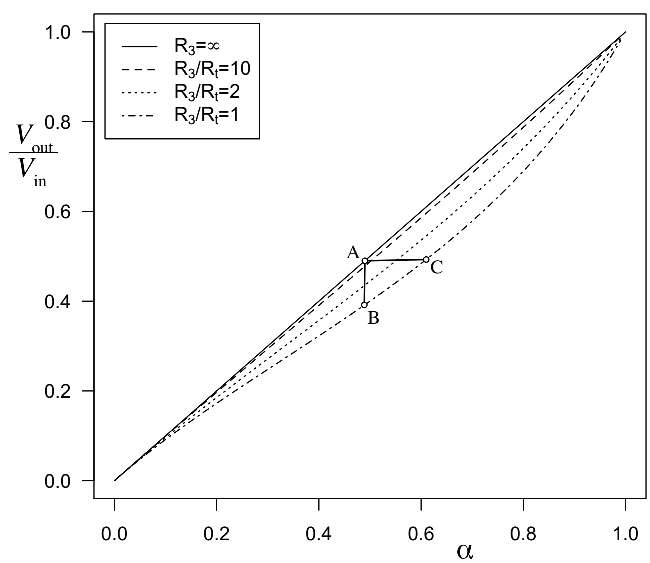


Figure 6: Influence of non-zero load resistor, R_3, on the output of the potentiometer circuit.

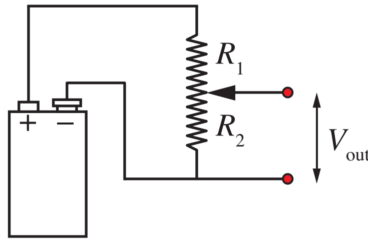


Figure 7: Using a voltage divider to supply variable voltage from a 9V battery.

3 Application: Using a 9V Battery to Supply 5V Power

Consider the circuit in Figure 7 in which a potentiometer is connected to a nine volt battery. By adjustment of the potentiometer, the circuit in Figure 7 allows a 9V battery to supply a voltage between 0 and 9V. It is not an efficient way to control the power voltage since power is dissipated in R_1 with no useful gain. However, this is one simple situation where a potentiometer *could* be used.

Suppose you wanted to create a 5V power supply using the idea from Figure 7, but you wanted to use fixed resistors instead of a potentiometer. Neglecting the current to the load resistor, what values of R_1 and R_2 would give $V_{\text{out}} = 5\text{V}$ if $V_{\text{in}} = 9\text{V}$?

We begin the analysis by rearranging Equation (5). Solve for R_1/R_2:

$$\frac{V_{\text{out}}}{V_{\text{in}}} = \frac{R_2}{R_1 + R_2} \quad \Longrightarrow \quad \frac{V_{\text{out}}}{V_{\text{in}}} = \frac{1}{\dfrac{R_1}{R_2} + 1} \quad \Longrightarrow \quad \frac{R_1}{R_2} + 1 = \frac{V_{\text{in}}}{V_{\text{out}}}$$

$$\therefore \quad \frac{R_1}{R_2} = \frac{V_{\text{in}}}{V_{\text{out}}} - 1 \tag{18}$$

Equation (18) allows us to find a ratio of resistors that will achieve a desired voltage output. For example, with $V_{\text{in}} = 9\text{V}$ and $V_{\text{out}} = 5\text{V}$ we obtain

$$V_{\text{in}} = 9\text{V}, \quad V_{\text{out}} = 5\text{V} \quad \Longrightarrow \quad \frac{R_1}{R_2} = \frac{9}{5} - 1 = \frac{4}{5}$$

What values of standard resistance satisfy the constraint $\dfrac{R_1}{R_2} = \dfrac{4}{5}$? Table 2 gives some standard values of 5% accurate resistors. Inspecting combinations of resistors shows that three possible combinations of resistors satisfy the requirement: $R_1 = 12\,\Omega$ and $R_2 = 15\,\Omega$, or $R_1 = 16\,\Omega$ and $R_2 = 20\,\Omega$, or $R_1 = 24\,\Omega$ and $R_2 = 30\,\Omega$.

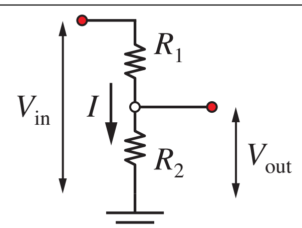
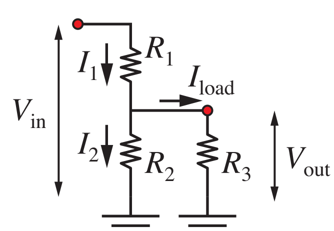
4 Summary

Table 3 shows the voltage divider schematics and output formulas for the two cases analyzed in Section 2. For most applications, we can safely use the simple formula obtained by assuming $R_3 \approx \infty$.

Table 2: Standard resistor values for 5% accuracy. Resistors are mass produced at powers of 10 times the base values listed on the left. The values on the right are an example of the base values times 100. See `http://www.rfcafe.com/references/electrical/resistor-values.htm`.

Base values	Base values $\times 100$
10, 11, 12, 13, 15, 16,	1000, 1100, 1200, 1300, 1500, 1600,
18, 20, 22, 24, 27, 30,	1800, 2000, 2200, 2400, 2700, 3000,
33, 36, 39, 43, 47, 51,	3300, 3600, 3900, 4300, 4700, 5100,
56, 62, 68, 75, 82, 91	5600, 6200, 6800, 7500, 8200, 9100

Table 3: Summary of voltage divider formulas. These formulas also apply when the two fixed resistors are replaced by potentiometer.

<table>
<tr><th>Case</th><th>Schematic</th><th>Formula for V_{out}</th></tr>
<tr><td>$I_{\text{load}} = 0$</td><td>
</td><td>$V_{\text{out}} = V_{\text{in}} \frac{R_2}{R_1 + R_2}$</td></tr>
<tr><td>$I_{\text{load}} \neq 0$</td><td>
</td><td>$V_{\text{out}} = V_{\text{in}} \frac{R_2}{R_1 \left(\frac{R_2}{R_3} + 1 \right) + R_2}$</td></tr>
</table>

5 Appendix A: Algebra to Simplify $1 - R_1/R_{\text{eff}}$

In this section we will obtain a simplified form of the expression $1 - R_1/R_{\text{eff}}$ that appears on the right hand side of Equation (11). Start by working with the ratio $R_1 R_{\text{eff}}$

$$\frac{R_1}{R_{\text{eff}}} = \frac{R_1}{R_1 + \dfrac{R_2 R_3}{R_2 + R_3}} = \frac{R_1(R_2 + R_3)}{R_1(R_2 + R_3) + R_2 R_3}$$

$$= \frac{R_1 \left(\dfrac{R_2}{R_3} + 1 \right)}{R_1 \left(\dfrac{R_2}{R_3} + 1 \right) + R_2}$$

Now, subtract the preceding result from 1:

$$1 - \frac{R_1}{R_{\text{eff}}} = 1 - \frac{R_1 \left(\dfrac{R_2}{R_3} + 1 \right)}{R_1 \left(\dfrac{R_2}{R_3} + 1 \right) + R_2} = \frac{R_1 \left(\dfrac{R_2}{R_3} + 1 \right) + R_2 - R_1 \left(\dfrac{R_2}{R_3} + 1 \right)}{R_1 \left(\dfrac{R_2}{R_3} + 1 \right) + R_2}$$

$$= \frac{R_2}{R_1 \left(\dfrac{R_2}{R_3} + 1 \right) + R_2}$$