ME 120

Practice problems: compute V, I, P in simple circuits

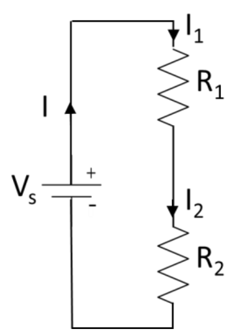
For all the problems shown below:

- R_i is the resistor number i, with i=1, 2, 3…
- R_{eq} is the equivalent resistor of the whole circuit
- V_i is the voltage across each resistor
- V_s is the voltage supplied by the battery (or voltage supply)
- I_i is the current going through each resistor
- I is the current leaving the battery
- P_i is the power dissipated by each resistor
- P_s is the power generated by the battery. Whatever the circuit, we always have: $\boldsymbol{P_s} = \sum \boldsymbol{P_i}$

Compute V, I, P for each component of each circuit **as a function of V_s and the resistors R_i**.

Hint: To compute the power in a timely manner, although $P_i = V_i . I_i$ you only need to calculate either V_i or I_i (whatever is the easiest), as the power can also be expressed as: $P_i = R_i . {I_i}^2$ or $P_i = \frac{{V_i}^2}{R_i}$ using Ohm's law and substituting in the equation.

1. 2 resistors in series



$R_{eq} = R_1 + R_2$

a) Let's compute the current through each component:

First, let's find I. We need to express the current as a function of V_s and the resistors R_i. Using Ohm's law across the battery, $V_s = R_{eq}.I$

Therefore $I = \frac{V_s}{R_{eq}} = \frac{V_s}{R_1+R_2}$

$I = I_1 = I_2$ .Therefore $I_1 = I_2 = \frac{V_s}{R_1+R_2}$

b) Let's compute the voltage across each component:

We already have V_s, as it is a given. Now, to find V_i across each resistor:

Using Ohm's law for each resistor ($V_i = R_i.I_i$) and substituting for I_i using the equation in a), we have:

$V_1 = R_1.I_1 = R_1.I = R_1.\frac{V_s}{R_1+R_2}$ and

$$V_2 = R_2.I_2 = R_2.I = R_2.\frac{V_s}{R_1+R_2}$$

We can check that we have Kirchoff's voltage law: $V_s = V_1 + V_2$

c) Let's compute the power generated by the battery and the power dissipated by each component:

Power generated by the battery: $P_s = V_s.I = V_s.\frac{V_s}{R_1+R_2} = \frac{{V_s}^2}{R_1+R_2}$

Power dissipated by each resistor:

We can express the power for each resistor in different ways using P=VI and V=RI.

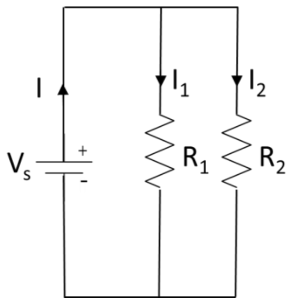
For R_1: $P_1 = V_1.I_1 = R_1.{I_1}^2 = R_1.I^2$

We have already computed *I*, and we find: $P_1 = R_1.\left(\frac{V_s}{R_1+R_2}\right)^2$.

Using the same logic, $P_2 = R_2.\left(\frac{V_s}{R_1+R_2}\right)^2$.

We can now verify that $P_s = P_1 + P_2$

2. 2 resistors in parallel



$$R_{eq} = \frac{1}{\frac{1}{R_1}+\frac{1}{R_2}} = \frac{R_1.R_2}{R_1+R_2}$$

a) Let's compute the voltage across each component:

$V_s = V_1 = V_2$

b) Let's compute the current through each component:

For the resistors, using Ohm's law for each component ($I_i = \frac{V_i}{R_i}$) and substituting for V_i using the equation in a), we have:

$I_1 = \frac{V_1}{R_1} = \frac{V_s}{R_1}$

$$I_2 = \frac{V_2}{R_2} = \frac{V_s}{R_2}$$

For the battery: $I = \frac{V_s}{R_{eq}} = V_s.\frac{R_1+R_2}{R_1.R_2} = V_s.\left(\frac{1}{R_2}+\frac{1}{R_1}\right)$

We can check that we have Kirchoff's current law: $I = I_1 + I_2$

c) Let's compute the power generated by the battery and the power dissipated by each component:

Power generated by the battery: $P_s = V_s.I = V_s.V_s.\left(\frac{1}{R_2}+\frac{1}{R_1}\right) = {V_s}^2.\left(\frac{1}{R_2}+\frac{1}{R_1}\right)$

Power dissipated by each resistor:

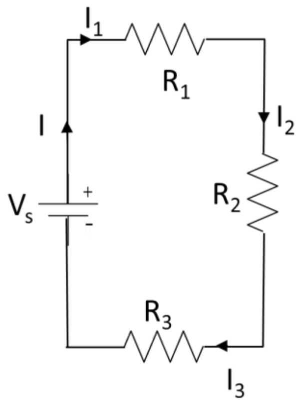
We can express the power for each resistor in different ways using P=VI and V=RI.

For R_1: $P_1 = V_1.I_1 = \frac{{V_1}^2}{R_1} = \frac{{V_s}^2}{R_1}$.

Using the same logic, $P_2 = V_2.I_2 = \frac{{V_2}^2}{R_2} = \frac{{V_s}^2}{R_2}$. .

We can now verify that $P_s = P_1 + P_2$

3. 3 resistors in series



$R_{eq} = R_1 + R_2 + R_3$

a) Let's compute the current through each component:

First, let's find I. We need to express the current as a function of V_s and the resistors R_i. Using Ohm's law across the battery, $V_s = R_{eq}.I$

Therefore $I = \frac{V_s}{R_{eq}} = \frac{V_s}{R_1+R_2+R_3}$

$I = I_1 = I_2 = I_3$ .Therefore $I_1 = I_2 = I_3 = \frac{V_s}{R_1+R_2+R_3}$

b) Let's compute the voltage across each component:

We already have V_s, as it is a given. Now, to find V_i across each resistor:

Using Ohm's law for each resistor ($V_i = R_i.I_i$) and substituting for I_i using the equation in a), we have:

$V_1 = R_1.I_1 = R_1.I = R_1.\frac{V_s}{R_1+R_2+R_3}$

$V_2 = R_2.I_2 = R_2.I = R_2.\frac{V_s}{R_1+R_2+R_3}$

$V_3 = R_3.I_3 = R_3.I = R_3.\frac{V_s}{R_1+R_2+R_3}$

We can check that we have Kirchoff's voltage law: $V_s = V_1 + V_2 + V_3$

c) Let's compute the power generated by the battery and the power dissipated by each component:

Power generated by the battery: $P_s = V_s.I = V_s.\frac{V_s}{R_1+R_2+R_3} = \frac{{V_s}^2}{R_1+R_2+R_3}$

Power dissipated by each resistor:

We can express the power for each resistor in different ways using P=VI and V=RI.

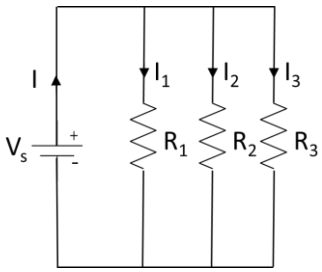
For R_1: $P_1 = V_1.I_1 = R_1.{I_1}^2 = R_1.I^2$

We have already computed I, and we find: $P_1 = R_1.\left(\frac{V_s}{R_1+R_2+R_3}\right)^2$.

Using the same logic, $P_2 = R_2.\left(\frac{V_s}{R_1+R_2+R_3}\right)^2$ and $P_3 = R_3.\left(\frac{V_s}{R_1+R_2+R_3}\right)^2$

We can now verify that $P_s = P_1 + P_2 + P_3$

4. 3 resistors in parallel



$$R_{eq} = \frac{1}{\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}}$$

a) Let's compute the voltage across each component:

$V_s = V_1 = V_2 = V_3$

b) Let's compute the current through each component:

For the resistors, using Ohm's law for each component ($I_i = \frac{V_i}{R_i}$) and substituting for V_i using the equation in a), we have:

$I_1 = \frac{V_1}{R_1} = \frac{V_s}{R_1}$

$I_2 = \frac{V_2}{R_2} = \frac{V_s}{R_2}$

$I_3 = \frac{V_3}{R_3} = \frac{V_s}{R_3}$

For the battery: $I = \frac{V_s}{R_{eq}} = V_s.\left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}\right)$

We can check that we have Kirchoff's current law: $I = I_1 + I_2 + I_3$

c) Let's compute the power generated by the battery and the power dissipated by each component:

Power generated by the battery: $P_s = V_s.I = V_s.V_s.\left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}\right) = V_s{}^2.\left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}\right)$

Power dissipated by each resistor:

We can express the power for each resistor in different ways using P=VI and V=RI.

For R_1: $P_1 = V_1.I_1 = \frac{V_1{}^2}{R_1} = \frac{V_s{}^2}{R_1}$.

Using the same logic, $P_2 = \frac{V_s{}^2}{R_2}$ and $P_3 = \frac{V_s{}^2}{R_3}$.

We can now verify that $P_s = P_1 + P_2 + P_3$